Alpha Duties:
The Search For Excess Returns and Appropriate Fiduciary Duties

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Abstract: It is standard investment advice to buy mutual funds or ETFs that are (i) well-diversified, (ii) low-cost, and (iii) expose one’s portfolio to age-appropriate stock-market risk. But advisors at times deviate from this advice in order for their clients to cash in on investment opportunities with expected above-market returns. For example, an advisor might have a client forego the benefits of diversification to invest in a stock or a handful of stocks that the advisor believes will produce “alpha,” excess returns. This article provides the first estimates of how large expected alpha has to be to justify sacrifices in diversification, cost and age-appropriate exposure. For example, we estimate that a person of average risk aversion would need an annual alpha between 500 and 1500 basis points before being willing to forego the benefits of diversification and hold an individual stock (and that during a financial crisis a person would need an alpha between 900 and 1800 basis points to make an alpha play utility enhancing). We propose that fiduciaries should be informed about these alpha tradeoffs and explain them to their clients before recommending (or executing) investments that deviate from the low-cost, well-diversified, age-appropriate exposure standard.

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I. INTRODUCTION

Investors and their financial advisors often try to beat the market by finding investments that pay above-market returns. This search for these higher returns is often referred to as “seeking alpha.” People are sometimes willing to give up diversification in order to seek supra-competitive expected returns. But few people understand how much alpha is necessary to outweigh the cost of not being fully diversified. We have calculated that for a person of average risk aversion would need to expect an annual alpha between 6 and 15 percent before being willing to forego the benefits of diversification and hold an individual stock (and that during a financial crisis would need to expect an alpha between 9 and 18 percent to make investing in a single stock utility enhancing). Such alpha would easily more than double the risk-premia normally paid on stock.

Of course not all alpha opportunities are so extreme as to necessitate investing solely in an individual stock. But some diversification is frequently sacrificed when investors adopt an alpha seeking strategy. Indeed, it is impossible to seek alpha without taking on some amount of idiosyncratic risk—because the very choice to overweight a portfolio toward an alpha opportunity implies some movement away from the portfolio that would have best diversified risk.

In this Article, we identify two other benefits that alpha investors at times sacrifice in their attempts to achieve above-market returns. Besides sacrificing the benefits of diversification, investors also at times take on too much or too little exposure to stock market risk when pursuing alpha investment opportunities. The diversification tradeoff involves taking on non-optimal amount of idiosyncratic (or diversifiable) risk, while the exposure tradeoff involves taking on non-optimal amounts of systemic risk. Some strategies involve both of these trade-offs. For example, an investor who believes her company will outperform the market and chooses to invest all her savings in it, might be exposed to non-optimal amounts of both systemic and idiosyncratic risk. Finally, investors may be willing to pay large fees to fund managers who they expect will deliver returns that more than offset the fee expense. Common-sense tells us that a manager charging a large, supra-competitive fee must obtain alpha of at least the size of the excess fee to make it worth investing with her. But intuition provides no clear guideline for what minimum alpha is required to justify sacrificing diversification or optimal exposure.

This Article makes two core contributions. First, it estimates the minimum offsetting alpha that would be required for investors to take on non-optimal amounts of systemic and idiosyncratic risk. Second, the Article explains how fiduciary duties should take into account these alpha tradeoffs. Our results have both general implications which apply across a variety of fiduciary contexts, and more specific applications for trustees, investment advisors, and 401(k) administrators.

1 The term “alpha” derives from the generic term for the intercept in the linear capital asset pricing model equation. See infra at text accompanying note 27.
Our goal in this article is to make alpha investing “safe, legal and rare.”² We do not propose that fiduciaries eschew all alpha opportunities, by insisting for example that portfolios be invested in low-cost passively-managed index funds.³ Rational investors, guided or unguided by fiduciaries, might at times identify a credible alpha opportunity. We make no claim that such risk-justified opportunities are fleetingly small. As a theoretical matter, there can be both Type I (mistakenly pursuing alpha which will not pan out) and Type II (mistakenly failing to pursue alpha which would deliver superior returns) errors with regard to alpha opportunities. And while some of our regulatory proposals might reduce Type II errors (for example, by enabling currently-chilled trust fiduciaries to more easily trade-off diversification for alpha)⁴ the bulk of our efforts here are to reduce Type I errors in the current equilibrium. Indeed, the very magnitude of required returns that we estimate below provide good reason for thinking that too many fiduciaries currently “seek alpha” on behalf of their clients.

Accordingly, we argue that fiduciaries who recommend or invest in alpha portfolios should be required to explicitly consider the costs as well as the benefits of seeking alpha. Specifically, fiduciaries should (i) substantively estimate the costs of excessive fees, failing to diversify, deviating from otherwise would be optimal exposure, (ii) separately estimate and justify the expected alpha from the investment decision, and (iii) show that the expected alpha benefits exceed the alpha-required to offset these associated costs. Fiduciaries who are recommending alpha-contingent portfolios should have a duty of explaining the pertinent tradeoffs to their clients. Moreover, fiduciaries should have dynamic mechanisms in place to update their recommendations based on evolving market conditions and to keep track at their success across clients with regard to predicting alpha.

Beyond this general duty to explicitly consider and explain alpha tradeoffs, our results have a number of specific implications for various financial fiduciaries. For example, our estimates suggest that the trustee’s duty to diversify trust investments should be stricter during periods of market upheaval because the value of diversification appears to increase during those periods. Likewise, we argue that, when a trust is explicitly permitted or even required to hold a concentrated position in the family business, the trustee and the courts must still be more sensitive to questions about whether to nevertheless diversify the trust during periods when idiosyncratic risk is high. To help assure that fiduciaries have the requisite training to implement an alpha cost-benefit assessment, we also recommend that FINRA licensing tests for broker-

² Versions of the quoted phrase have been used by, among others, Bill Clinton. Speech to the Democratic National Convention (Aug. 29, 1996) (“Abortion should not only be safe and legal, it should be rare.”). The suggestion that government should reduce the prevalence of abortion is contested, in part because it threatens to impinge on a fundamental right to privacy. The idea that to protect investors the law should discourage fiduciaries from recommending (or executing) strategies which cut against a consensus among economists and financial professionals, is far less controversial.


⁴ See infra at text accompanying note 64.
dealers and certified investment advisors be enhanced to assure that fiduciaries are cognizant of the three tradeoffs at the heart of our analysis. Finally, we suggest that ERISA be revamped to reduce the chance that self-directed retirement account make ill-advised alpha investments. Specifically, we propose that the Department of Labor should issue new regulations interpreting §404(c). These regulations would require that, in order to qualify for safe-harbor immunity, 401(k) plan sponsors periodically provide investors with an individualized portfolio analysis of potential diversification, exposure, and fee mistakes. This disclosure, which can be provided at minimal cost in this new age with the advent of fin-tech robo-advisors, should include the warnings about the requisite alpha that would be required justify the participants portfolio choices. We also suggest enhanced disclosure by plan sponsors to the Department of Labor so that the department can better track the extent to which participants implicitly seek alpha and how often they succeed. Last, if the ERISA “fiduciary rule” is either pared back or repealed, we suggest that fin-tech disclosures about required alpha could play a helpful role in reducing the influence of financial advisers who, consciously or unconsciously, steer clients toward mutual funds which pay the adviser larger commissions. In particular, these funds tend to be higher fee, and under-diversified and a clear disclosure of how these infrequently the funds beat the market by enough to justify these costs might dissuade clients from taking up this conflicted advice.

The remainder of this article is divided into three Parts. Part II explains theoretically why alpha expectations might justify what otherwise would be failures to diversify, to minimize fees, or maintain age-appropriate exposure to equities. Part III presents our empirical estimates of the alpha required under a variety of conditions – including lumpy vs. non-lumpy investment opportunities. We show that the minimal requisite alpha is substantially higher in periods of financial crisis than in non-crisis times, and how this requisite changes with different levels of risk aversion and different degrees of diversification loss. Finally, Part IV draws out the normative implications of our analysis for three different sets of investment fiduciaries: trustees who might pursue alpha opportunities when investing trust assets, FINRA fiduciaries (broker/dealers and investment advisors) who might recommend or execute alpha opportunities for their clients, and ERISA fiduciaries who might offer alpha opportunities in the plan menus.

II. DISTINGUISHING BETWEEN MISTAKES AND TRADEOFFS

A. The Three Central Investment Mistakes

While the question of how best to invest non-precautionary savings is often viewed by retail savers as daunting, the consensus is actually easily described: Absent some opportunity to beat the market, it is best to invest in vehicles (such as mutual funds or ETFs) that are (i) well-

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6 In fact, one of us has described it while standing on one-leg. See Ian Ayres, Concise Advice For Investing, While Standing On One Leg, FORBES, (2016), available at https://www.forbes.com/sites/whynot/2016/01/30/hillel-investment-advice/#60501a4b6b55. It might be accomplished by as simply as investing your assets in a Vanguard target date fund with the date you plan to retire.
diversified, (ii) low-cost, and (iii) expose one’s portfolio to age-appropriate stock-market risk. The flip-side of this guidance is that there are three central investment mistakes: failing to diversify, paying high (supra-competitive) fees, and failing to expose one’s portfolio to an appropriate amount of market risk.

Failing to diversify can be an investing mistake, because diversification allows investors to reduce the volatility of returns without reducing expected returns. As a theoretical matter, full diversification would require portfolios holding some of every risky asset—including, for example, international equities, real estate investments and all manner of fixed income securities.7 In practice, substantial benefits from diversification can be achieved by holding as few as 10 well selected large cap stocks.8 While a portfolio of this size is far less risky than a single-stock portfolio, there remain very important benefits to further diversification, particularly during periods of high volatility.

We estimate that diversification during normal times can reduce the standard measure of volatility, the standard deviation of the annual return, by 14.3%--from 33.5% on an average individual U.S. equity to 19.2% on a fully diversified portfolio U.S. stocks.9 What’s more, the benefits of diversification tend to be greater during the crisis periods. In Figure 1, we plot the standard deviation of a diversified portfolio of CRSP stocks and the average volatility of individual stocks over time. During times of crisis, the diversification reduces the standard deviation of return, by 16.6%--from 51.4% on an average individual stock to 34.8% on a diversified portfolio of U.S. stocks. Failures to diversify risk are often not as stark as investing all of your savings in company stock, but lower bound estimates on partial failures to diversify 401(k) savings have been estimated to be equivalent paying excess fees annually of 0.71%.10

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9 These figures are drawn from a “Monte-Carlo simulation” which uses historical stock data to build a distribution of returns for both a diversified portfolio and the “average” individual stock. Put differently, we look to historical data to estimate the probability that an average individual stock or a diversified portfolio whole will rise by 25% or fall by 25% in a year, rise by 26% or fall by 26% etc. Specifically, we use monthly data on stocks traded on the NYSE, AMEX, and NASDAQ available from the Center for Research in Securities Prices (hereinafter “CRSP”) from 1926 to 2015. We split the sample into “normal” periods, and “crisis” periods, by looking at the volatility of the market over the prior 30 days. If the annualized market volatility over the past 30 days was over 25%, then we classify the month as a crisis period.

To construct the average individual stock distribution during normal periods, we randomly draw (with replacement) 100,000 individual stocks from across the normal months—where the probability of selecting a stock is proportional to its share of the total market capitalization during that month—and calculate the return of that stock for the following year. We do the same for crisis months. Likewise, we follow a similar procedure for the diversified portfolio, where the diversified portfolio is a market-cap weighted average of all the U.S. stocks.

10 See Ian Ayres & Quinn Curtis, Beyond Diversification: The Pervasive Problem of Excessive Fees and “Dominated Funds” in 401(k) Plans, 124 YALE LAW REV. 1479, 1500 (2015) (Table 1).
Paying excessive fees can be an investment mistake, because these fees eat away at the net return. For example, paying an excess fee of 2% over time can halve your retirement savings. Overcharges on this order of magnitude have routinely occurred in the real world. One of us, in analyzing more 3,500 401(k) plans (with more than $120 billion in assets), found that the top 5% had excess fees of 2.05% (with the average excess fees of .63%).

Exposing one’s portfolio to the wrong amount of market risk is a mistake because investors who take on too much or too little stock market risk fail to optimally tradeoff risk and return. We will henceforth call this a “beta” mistake because in the Capital Asset Pricing Model (CAPM) beta is a numeric measure of how exposed a portfolio is to market risk. A portfolio with a beta of zero is invested in risk-free assets, while a portfolio with a beta of one is invested

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11 $10,000 invested at a 6% annual return over 40-years will come to equal $102,857, but at 4% return will yield only $48,010. See [http://www.npr.org/2015/10/30/453163154/when-high-fees-stink-up-your-401-k-what-you-can-do](http://www.npr.org/2015/10/30/453163154/when-high-fees-stink-up-your-401-k-what-you-can-do)

12 See I. AYRES & Q. CURTIS, supra note 10, at 1500 (Table 1). 63 basis points of excess fees would reduce the 40-year nest-egg by more than 20% (from $102,857 to $81,036). See supra note 11.
100% in equities. Investors can make beta mistakes by exposing their portfolio to either too much or too little stock market risk given their personal risk tolerance. The two types of beta mistakes are depicted in the following figure:

Point A in Figure 2 depicts the expected return and risk (standard deviation of expected return) of a portfolio that optimally balances risk and return for a particular investor. In this Figure, the straight-line is the “Capital Allocation Line,” which represents the set of the best achievable investment portfolios. (These are the best portfolios because in a simple CAPM model like this, one cannot beat the market). Each point on the Capital Allocation Line is uniquely associated with a particular beta—that is the percent of the portfolio exposed to market risk. At the far left (the Y-axis) the portfolio is composed exclusively of risk-free assets, which earns the risk-free rate, \( R_f \). Because it has no market exposure, this portfolio thus has a beta of 0. The beta increases as one moves along the Capital Allocation Line to the northeast (say, from point B to C). The curved lines represent this investor’s “iso-utilities,” the set of returns and risks for which the investor’s utility is constant. Higher iso-utility curves lie northwest because investors prefer higher expected returns and lower risk. Point A is optimal because at that point the benefits to the investor of decreasing risk by moving down the Capital Allocation Line are exactly offset by

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13 Under the standard CAPM model, there is a single fully diversified portfolio of risky assets—the market portfolio—which all investors will own. Investors, however, will vary what proportion of their wealth they hold in the market and the Capital Allocation Line represents the return and risk of different combinations of risk-free assets and the market portfolio.

The model does not account for frictions like transactions costs, taxes, the inability of investors to borrow at the risk-free rate, the lack of full information available to all investors, and assumes homogenous investor expectations.
the value she places on the associated decrease in expected return (and vice-versa moving up the CAL).\textsuperscript{14} Points B and C depict exposure mistakes with portfolios that place the investor on a lower iso-utility curve. Point B represents a portfolio that includes too few risky investments, given the investor’s risk preferences, while Point C represents a portfolio that includes too much risky investment.\textsuperscript{15}

Richard Merton in 1969 offered a simple equation to estimate the optimal portfolio exposure as a function of just three variables:\textsuperscript{16}

$$\beta_M = \frac{\text{Risk Premium}}{\sigma^2 \ast \text{Risk Aversion}}$$

where $\beta_M$ is the measure of optimal exposure to market risk, the Risk Premium is the amount by which the return on risky assets (say, a diversified portfolio of stocks) is expected to exceed the risk-free return (on say, government bonds), $\sigma^2$ is the expected volatility of returns (captured, say by the variance of expected stock returns) and Risk Aversion is the investor’s “relative risk aversion” which measures how sensitive she is to risk, with 0 indicating she is risk neutral and with larger numbers indicating an increasing unwillingness to bear additional risk to get a fixed increase in her expected returns.\textsuperscript{17} Like many economics models, Merton’s assumes that investors exhibit “constant relative risk aversion,” (“CRRA”) which much empirical work, though not all, suggests this is a reasonable approximation of real behavior.\textsuperscript{18} We also adopt the assumption of constant relative risk aversion in our empirical work below.\textsuperscript{19} Studies estimate

\textsuperscript{14} Formally, Point A optimally trades off risk and return because at this tangency, the investor’s marginal rate of substitution (the rate at which consumer is ready to give up a higher expected return in exchange for reducing risk while maintaining the same level of utility) equals the market’s marginal rate of transformation (the rate at which return must be sacrificed in order to produce a marginal unit of reduced risk). The figure is the CAPM equivalent of microeconomics consumption graphs, with the capital market line being analogous to the production possibility frontier or budget set.

\textsuperscript{15} At point B, the investor’s marginal rate of substitution exceeds the marginal rate of transformation – meaning that move slightly to the northeast along the Capital Allocation Line would increase the investor’s utility, because the investor at point B would be willing to take on the additional risk in order to achieve a higher expected return. The reverse is true at point C.


\textsuperscript{17} Returning to Figure 2, the investor’s relative risk aversion is technically a measure of the curvature of her iso-utility curve.


\textsuperscript{19} Constant relative risk aversion is often used in conjunction with CAPM, even though for CAPM to exactly predict prices in the model, investors must have quadratic utility. Our use of CAPM should therefore be thought of as a simple way to approximate how returns are generated. See P. Jean-jacques Herings & Felix Kubler. Approximate CAPM When Preferences Are CRRA, 29 COMPUTATIONAL ECONOMICS (2007).
that the relative risk aversion of average investors is in the range of 2 to 4. For example, if the risk premium is 4%, the standard deviation is 20% and risk aversion is 2, then the optimal beta will be 50%. 

Merton’s investment exposure equation makes intuitive sense: an investor should, all else equal, be willing to hold a portfolio that is more exposed to market risk when the expected premium of holding risky assets is larger, and be less willing to hold a portfolio that is more exposed to marked risk when the expected volatility of risky assets is higher or if the investor is more averse to that risk.

Merton’s exposure equation, however, excludes the age of investor. If investors tend to become more risk-averse as they age, then it would be natural that they would reduce their equity exposure as they grew closer to retirement. Target date mutual funds tend to follow a variety of age-contingent strategies, such as the following “birthday rule”:

$$\beta_B \approx 110 - \text{investor age}$$

A target-date fund following the birthday rule would invest approximately 90% of its assets in equities when the investor is 20 and approximately 50% of assets in equities when the investor is 60. While there isn’t a well-known term to capture the possibility of exposure or beta mistakes, the possibility is indirectly captured by ERISA regulations regarding default investments options when a 401(k) participant fails to provide investment instructions. One of the ways an investment product can qualify as a “Qualified Default Investment Alternative” in which those 401(k) funds can be invested is to have “a mix of investments that takes into account the individual’s age or retirement date” such as a target date retirement fund.

To assess whether an investor is making a beta mistake, we must know what the right exposure to stock market risk would be. Reasonable people can differ over some range of exposures. However, some exposures are prima facie unreasonable judged by any of these standards. For example, one study found that in 2007 roughly half of 401(k) participants in

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21 $$(.04)/(2*.2*2)=.5.$$.


23 Indeed one of us argued for a third “leveraged lifecycle” approach:

$$\beta_{LL} = \max \left[ 2, \frac{\beta_M \cdot PV(\text{Lifetime Savings})}{\text{Present Savings}} \right],$$

where $PV(\text{Lifetime Savings})$ is the risk-adjusted expected future and present savings and $\text{Present Savings}$ is the amount that the investor has currently saved. See IAN AYRES & BARRY NALEBUFF, LIFECYCLE INVESTING: A NEW
their 20s had no exposure to equity. These investors are likely making exposure mistakes (akin to Point B in Figure 2) by not capturing any of the substantial risk premium on equity.\textsuperscript{24} Such low beta portfolios fail both the Merton and Birthday Rule beta standards.\textsuperscript{25} Of course, with sufficiently high risk aversion or pessimistic market expectations, a low beta might be justified. But young people putting all their savings in money market accounts is a horrible way to save for retirement. The same study found that more than a fifth of older 401(k) participants (ages 56-65) had more than 90% of their portfolio in equities. This is likely an example of the second type of exposure mistake (akin to Point C in Figure 2), as these participants are arguably exposing too much of their assets to stock market risk. However, it is admittedly harder to empirically identify this second form of beta error. Oldsters who invest almost entirely in equities are inconsistent with the lifecycle dicta, but not necessarily inconsistent with Merton’s exposure equation, if, for example, the participants are not particularly risk-averse and hold more sanguine views about the stock market.\textsuperscript{26}

B. Three Alpha Investing Tradeoffs

While the last section explained how failing to diversify, economize on fees, or give one’s portfolio appropriate exposure to equities can be mistakes, this section instead explains how each of these deviations might instead be justified by sufficient expectations that particular investment opportunities will deliver risk-adjusted returns superior to investing in the market as a whole. We will call such opportunities “alpha” investments following popular finance parlance. The term derives from how one might measure whether an investment generates excess returns: regressing the returns of that investment on the returns of a diversified portfolio of risky assets, which we will henceforth simply call the “market portfolio.”

The regression in (simplified form\textsuperscript{27}) is: $R_{i,t} = \alpha_i + \beta_i (R_{m,t})$ where $R_{i,t}$ is the return of the investment in question in period $t$, $R_{m,t}$ is the return on the market portfolio in period $t$. If

\begin{align*}
\text{SAFE AND AUDACIOUS WAY TO IMPROVE THE PERFORMANCE OF YOUR RETIREMENT PORTFOLIO (2010). This leveraged-lifecycle equation, the authors offer does a better job of diversifying risk across time and reduces an investors exposure to the that of Merton equation as the investor approaches retirement (because PV(Lifetime Savings) will equal current savings when there are no future saving increments.}
\end{align*}


\textsuperscript{25} They are even more inconsistent with the Leverage Lifecycle standard proposed by Ian Ayres and Barry Nalebuff. See I. AYRES & B. NALEBUFF, supra note 23.

\textsuperscript{26} In addition, investors who have saved far more than they will personally consume during their lives might rationally invest more aggressively because they wish to pass along as much as possible to their heirs.

\textsuperscript{27} In fact, both the returns on the investment and the market should be measured net of the risk-free rate, and there must be an error term which adjusts for the fact that the returns of any investment are not perfectly predicted by the market in every period. Thus, the full regression is $R_{i,t} - R_f = \alpha_i + \beta_i (R_{m,t} - R_f) + \epsilon_{i,t}$, where $R_f$ is the risk-free rate and $\epsilon_{i,t}$ is the mean-0 error term.
the investment outperforms the market, the regression will yield a positive intercept, $\alpha$, hence the term “alpha.” Note that by controlling for the investment’s correlation with market returns, $\beta_i$, the regression adjusts for the investment’s exposure to market risk. Thus, alpha will not automatically be generated by investments with high market exposure and high expected returns.

Graphically, the possibility of an alpha investment is depicted in Figure 3:

**Figure 3**

In Figure 2, we assumed away a number of real world complexities, which meant that alpha opportunities were impossible. Recall that in that world, the Capital Allocation Line (CAL) represents the set of the best achievable portfolios, which consists of (0 alpha) portfolios mixing risk-free assets and the market portfolio (point M). If we now consider a world in which alpha opportunities can exist, alpha investments will lie above the CAL, like points A and B in Figure 3.\textsuperscript{28} Since point A is to the left of point M, it represents an investment with a $\beta < 1$. In contrast, point B represents an alpha investment (again lying above the CAL), but with more heightened exposure to systemic risk with a $\beta > 1$. Because the risk-adjusted expected returns of these two investments exceed the expected market return, one would rationally want to hold them as part of a diversified portfolio. Indeed, the excess returns could even cause one to be willing to overweight them in portfolio – investing more than would be necessary to diversify.

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\textsuperscript{28} Alpha investments might also be based on a privately held belief that the systemic (beta) risk of a particular stock is lower than the market (and beta regressions) suggest. An alpha investor would in this case want to tilt toward larger portfolio holdings of the stock not because it lies “above” the CAL given its beta, but because it lives to the “left” of the CAL. But this alternative characterization would continue to produce the same type of increased exposure to idiosyncratic risk indicated by the movement from point A to A’.
Overweighting an alpha opportunity will come at a cost, however. The investor will bear some of the risk specific to the alpha investment—its “idiosyncratic risk”—which would have been diversified away if she did not overweight it. To make this more concrete, imagine that the alpha investment opportunity is “lumpy”: the investor must invest all her savings in A or buy none at all. (Say A is a startup with a minimum investment equal to the investor’s savings). The additional idiosyncratic risk of investing only in A is shown in Figure 3 as point A’ which lies horizontally to the right of A. Given its level of systemic risk, A is a positive alpha opportunity lying above the CAL, but once we account for the loss of diversification, such a lumpy alpha opportunity need not make the investor better off. Indeed, as shown in Figure 3, point A’ lies below the CAL.

More generally, the additional expected return from investing in a lumpy alpha opportunity might or might not exceed the detrimental loss of diversification. For example, Figure 4, shows three possible outcomes of bearing the idiosyncratic risk of an alpha opportunity.

**Figure 4**

Point B’ is, like point A’, an alpha opportunity that lies below the CAL once we account for idiosyncratic risk. Point B’’ is an alpha opportunity that lies above the CAL once we account for idiosyncratic risk, but still lies below the utility the investor could achieve by investing in a fully diversified portfolio at point D. And Point B’’’ is an alpha opportunity that lies above both the CAL and the investor’s utility from holding a diversified portfolio. Only in the last case (Point B’’’) would the investor be better off foregoing the benefits of diversification and placing all her savings in the alpha opportunity. Accordingly, it is not true that investors should always remain diversified. But the foregoing shows that sacrificing diversification requires a sufficient offsetting alpha.
A similar argument also applies to an investor trading off alpha for moving away from her ideal beta (exposure to market risk). A lumpy alpha opportunity might force an investor to be exposed to more or less market risk than she would have chosen from the zero-alpha alternatives on the CAL. As discussed above, this deviation in exposure would reduce the investor’s expected utility. But, as before, expecting a sufficient alpha can outweigh the costs of departing from the ideal level of exposure. For example, in Figure 5, point D reflects the optimal portfolio for an investor, absent any alpha opportunities.

**Figure 5**

![Figure 5](image)

Now imagine that the investor is offered an alpha investment that exposes her to less systemic risk than at point D, which means she will also obtain less of the risk premium. This positive alpha investment might either be utility enhancing or not. In Figure 5, Point A’ shows an alpha opportunity that lies above the CAL but not above the investor’s iso-utility curve. Point A’’ in contrast shows an alpha opportunity that lies above both the CAL and the investor’s iso-utility curve. Only in the latter case would the alpha benefit outweigh cost of having a beta which is too low given the investor’s risk preferences.

Finally, the opportunity to obtain alpha might justify paying what otherwise would seem to be excessive fees. For example, imagine an investor is contemplating whether to invest in a well-diversified, actively managed mutual fund that charged $f$ basis points in fees more than competitive passive indexes, but which is expected to generate excess returns of $\alpha$ basis points. Here, the fee/alpha tradeoff is relatively straight-forward. The key question is whether the excess expected returns justify the excess management fees:

$$\alpha > f$$

One should invest in the alpha opportunity only if expected alpha is greater than the excess fee. Graphically, this condition requires that the expected return of the opportunity *net of the excess fee* lie above the Capital Allocation Line.

This example again isolates a single tradeoff, here the fee/alpha tradeoff, because by assumption the actively managed fund is well diversified and non-lumpy so that the investor
need not take on idiosyncratic risk and can adjust his or her equity exposure by mixing the fund to different degrees with government bonds. The foregoing analysis has focused on isolating the impact of diversification, exposure, and fee tradeoffs. Real-world investments at times only require considering the tradeoffs on one of these three dimensions. A mutual fund focused on one industry might sacrifice diversification without sacrificing fees or exposure. Or a high-fee target date fund (such as Fidelity Freedom Funds with expense ratio as high as 70 basis points annually) might sacrifice competitive fees without diversification or exposure. Or a 20-year old’s 100% money-market portfolio investment might sacrifice exposure without sacrificing diversification or competitive fees. In each of these examples, an investor would need to have a sufficient alpha expectation to justify the isolated sacrifice of diversification, competitive fees or optimal equity exposure.

But in many other contexts the alpha investment opportunity will entail sacrificing some combination of diversification, competitive fees, or optimal market exposure. Actively managed funds, for example, usually have both higher fees and require some diversification sacrifices because the fund managers must pick a limited number of firms which they believe will outperform the market. Lumpy, all-or-nothing investment opportunities are particularly prone to simultaneously requiring the sacrifice of both diversification and optimal equity exposure. Starting a family business, for example, might both expose an investor to idiosyncratic risk and too much (or too little) systemic risk. The key question in such situations would be whether the alpha expectation is sufficient to justify the total risk (systemic and idiosyncratic combined) that the investor has to take on. As shown graphically, this means not only that the expected return lies above the Capital Allocation Line when considering total risk, but the expected return lies above the iso-utility curve for the next best market alternative.

Our theoretical analysis has focused on the Capital Asset Pricing Model and lumpy alpha opportunities, but it can easily be generalized. For example, Eugene Fama and Kenneth French have identified two attributes (or “factors”) that empirically have been associated with excess returns, namely firms with small market capitalizations and those with a high ratio book value to market value. From a CAPM perspective, the excess returns that tend to be garnered by small cap stock or high book value stock can be interpreted as an “alpha” which would lead rational investors to want to overweight small cap stock in their portfolios. But as depicted in Figure 5, this overweighting will cause at least some diversification loss, pushing up the portfolio risk, possibly inside the Capital Allocation Line (as in B’). Rational investors would not want, however, to overweight in ways that reduced utility below the iso-utility line.

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29 Available at https://fundresearch.fidelity.com/mutual-funds/summary/31617R704
30 Investing one’s entire portfolio in money market or government bonds might be considered an alpha opportunity – allowing one to beat the market – if one expected equity prices to fall.
31 Note that an investor would prefer a lumpy investment to have a lower beta than if she was investing in a diversified portfolio. Intuitively, this is because the addition of the lumpy investment’s idiosyncratic risk makes the investor more sensitive to risk in the tradeoff between systemic risk and expected return. It is akin to the investor having become more risk averse, and this means she will prefer a lower beta when confronting an investment with idiosyncratic risk.
How much to overweight becomes a central concern when the alpha opportunity is not a
lumpy, all-or-nothing investment choice, but can be chosen by an investor in various increments. An actively managed mutual fund with high management fees is a quintessential example of a non-lumpy investment because the investor can vary the proportion of her portfolio that she chooses to invest in the high-fee fund. An opportunity to start a family business in contrast is much more lumpy investment as it might require committing a substantial proportion of an investor’s portfolio. Minimum investment requirements imposed by various types of funds (including hedge and private equity funds) also can make investment options a lumpy or discrete portion of a portfolio.

Theory tells us that when a non-lumpy alpha opportunity arises rational investors would want to “tilt” or overweight their portfolios toward the investment. The extent of tilt will depend on the particular costs and benefits (and will be empirically estimated in the next section). When the alpha opportunity is lumpy, the optimal all-or-nothing investment choice will be “nothing” if the alpha is not sufficient to justify the incremental diversification, exposure, and fee losses.

III. Empiricism

The last Part explained as a theoretical matter why investment opportunities with expectations of sufficient above market returns could justify reduced diversification, inflated fees, or non-optimal exposure to market risk. In this Part, we turn from theory to numbers -- to estimate how much alpha is required to justify a failure to diversify, economize on fees, or obtain age-appropriate exposure to market risk. The estimates (and the ability to make such estimates) are important because, as we will argue in Part IV, fiduciaries who make one of the presumptive mistakes without considering whether they have the requisite alpha or who do not have a sufficient basis for believing that an investment opportunity has a sufficient alpha might, in a variety of contexts, be held liable.

A. Excess Fees

The required alpha to justify a mutual fund’s excess fees is the easiest to estimate. As mentioned in the last Part, the required alpha is simply the amount by which the fees exceed the competitive expense ratio charged by other funds offering well-diversified portfolios of similar investment classes. It would be a “nirvana fallacy” mistake to assume that the competitive market can offer diversified portfolios at zero cost. For domestic equities, there are a host of diversified funds and ETFs that annually charge less than 25 basis points, while for emerging markets the competitive expense ratios are somewhat more but many are offered with fees of less

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33 Intuitively at the optimal beta the investor is essentially indifferent between taking on marginally more risk and getting marginally more return by moving up the Capital Allocation Line. Purchasing a tiny amount of the alpha opportunity, however, provides a better trade-off between risk and return than moving along the Capital Allocation Line and hence all investors will want to purchase at least a tiny amount of the alpha opportunity if possible.
than 50 basis points. It is only the excess above the competitive price that needs to be traded off against alpha. When considering combination fee tradeoffs, one can begin by simply subtracting the excess fees from the expected alpha and then ask whether the alpha net of excess fees is sufficient to justify the short fall in diversification or exposure. Thus, in considering the required alphas estimated below, they should be construed as the net alphas that are required to take on deviations from optimal diversification or exposure.

Various studies have reported negative average mutual fund alphas (ranging between -0.43% and -0.68% per year). Nonetheless, some scholarship suggests that bets on actively managed higher fee mutual funds do exist for a small percentage (less than 3%) of funds. However, studies suggest that such alpha over-performance is not persistent. For example, a

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34 A list of funds with expense ratios listed is available at: https://money.usnews.com/funds/etfs/rankings/diversified-emerging-nkts.


recent study found that of the top half of funds in 2010, only 4.47% were able to stay in the top half for five years, and only 0.28% stayed in the top quarter.38

B. Diversification Costs

Estimating the required alpha to justify sacrificing diversification is the central empirical motivation for this article. Imagine that you had a lumpy choice of either investing all your savings in a single representative stock of public U.S. companies (say, your company’s) or in a fully-diversified mutual fund of U.S. equities. How big would the expected alpha on the single stock have to be to justify the obvious loss in reduced idiosyncratic risk that could achieve through diversification?39

To answer this question, we examine historical data on U.S. stocks from the mid-1920s through 2015. We calculated the utility of investors with various levels of risk aversion from holding either a diversified portfolio or a single stock over the course of one year. We then estimate how much alpha the individual stock must generate before an investor will prefer the individual stock with its higher alpha boosted returns but higher risk to the diversified portfolio. We made separate calculations for periods of market upheaval because idiosyncratic risk rises during economic crises,40 meaning that the required alpha will usually rise as well. We define these crisis periods as those in which the annualized standard deviation of market returns over the previous month was 25% or more. (Further details on our calculations are included below the table).


39 This comparison is close to an analogous choice between investing costlessly in a single representative stock or investing in a fully diversified mutual fund with excess fees. And the analogous question is: how high would the excess fees have to be before you would you would prefer investing in the single stock? This question arises in real world settings as some 401(k) plans offer company stock with no (or minimal) transaction or annual fees while offering mutual funds with annual expenses well above 100 basis points. See I. AYRES & Q. CURTIS, supra note 10, at 1502 (finding that 48% of plans offering company stock have such high fees that some investors would be wise to rationally sacrifice diversification by tilting their portfolio toward company stock investments).

Table 1
By How Much Must a Risk-Averse Person Outperform the Market to Make Holding a Single Stock Worthwhile

<table>
<thead>
<tr>
<th>CRRA Coefficient</th>
<th>Regular Period</th>
<th>Crisis Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA Coefficient = 1 (Log Utility)</td>
<td>2.90%</td>
<td>4.70%</td>
</tr>
<tr>
<td>CRRA Coefficient = 2</td>
<td>6.30%</td>
<td>9.50%</td>
</tr>
<tr>
<td>CRRA Coefficient = 3</td>
<td>10.40%</td>
<td>14.30%</td>
</tr>
<tr>
<td>CRRA Coefficient = 4</td>
<td>14.90%</td>
<td>18.40%</td>
</tr>
</tbody>
</table>

Notes: The holding period is one year. The distribution of both the market and single-stock returns is constructed by randomly sampling from the historical returns of firms in the CRSP database from 1926-2015 (i.e., by Monte Carlo simulation). For the market, we use the market-cap-weighted average of all of the CRSP firms. We then calculate the real 1-year return for each month for this market portfolio during 1926-2015, and then randomly select (with replacement) 100,000 of the 1 year returns to create a distribution. Similarly, we calculate a 1-year real return for every firm for each month during the period and then randomly select 100,000 firm-years. The probability of selection is proportional to the firm’s market cap as a percent of total market cap at the start of that month. "Crisis" periods are defined any month in which the annualized standard deviation of market returns in the previous month was greater than 25%.

Table 1 teaches several important lessons. First, we can see that rational investors, even during regular periods, would require quite substantial alphas before foregoing the benefits of diversification. For investors with moderate risk aversion (measured by CRRA’s between 2-4),

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41 Note that our assumptions bias the estimated alphas downward because we assume the alpha is guaranteed. In reality, investors will not know with certainty whether a stock they believe will outperform the market will in fact do so. Accounting for this uncertainty increases the required alpha for risk-averse investors, sometimes substantially. For example, if the actual alpha is uniformly distributed between 0 and 2 times the expected alpha, the required alpha for investors with CRRA of 4 to 21% in normal periods and 48% in crisis periods. These estimates are presented in the Technical Appendix, Figure A-1.
the required excess annual returns by which an investment would need to be expected to beat the market ranges from 6.3% to 14.9%.\footnote{As mentioned in note 39, these estimates are very close to answers for the analogous choice between investing in a high fee mutual fund versus a single (zero alpha) stock. In the latter case, the fees on the diversified mutual fund would need to exceed 500 basis points for many investors before they would prefer investing in a single stock to the high fee fund. These extremely high fees probably exceed the risk premium that could reasonably expected by investing in the mutual fund or the stock, however. Thus, these investors, if given the choice, would prefer to invest primarily in Treasury bonds rather than the 500 basis point fee fund or the single stock.} Intuitively, investors with higher levels of risk aversion demand greater increases in expected return to bear the same increase in risk. Thus, they will require a higher alpha before they are willing to bear the same amount of additional idiosyncratic risk.\footnote{Note that we are assuming that the investors put their entire savings into either the market or a single stock and have no other source of income. If an investor has outside income like a salary from a safe job, her total wealth/consumption will be less variable and therefore she will not demand quite as large alphas to bear additional idiosyncratic risk. The same is true if that outside income comes instead from the investor placing part of her portfolio in safe assets like inflation protected government bonds. If an investor keeps 20% of her wealth in safe assets, that reduces the alphas by about 20 to 25% depending on her level of risk aversion. On the other hand, most of us actually bear additional risk through our human capital; our salaries are not safe, but instead depend on how our firm, the industry in which we work, and the economy as a whole are performing. In such cases, risk averse investors might rationally demand even higher alphas to bear additional idiosyncratic risk by owning, say, company stock.} As we discuss in detail, these large alphas are consistent with the increasing emphasis on the importance of diversification in fiduciary law, particularly in trusts, over the last 30 years.

Second, we see from the table that the alpha required during crisis periods is substantially larger than during regular (non-crisis) periods. For investors with moderate risk aversion (again measured by CRRA’s between 2–4), the required alpha to forego diversification benefits ranges from 9.5% to a whopping 18.4%, ballooning during these crisis times. This is primarily because idiosyncratic risk rises during crises. During ordinary periods, the idiosyncratic risk (measured as a standard deviation of return) is 27.4%, while during crisis periods the standard deviation is 37.8%. As idiosyncratic risk increases, the benefits of diversification increase and therefore the alpha required to get investors to give up diversification increases as well.\footnote{In addition, crisis periods are defined by increases in systemic risk: during ordinary periods, the systemic market risk (measured as a standard deviation of expected return) is 19.2%, while during crisis periods the standard deviation is 34.8%. This rise in systemic risk increases risk-averse investors sensitivity to taking on additional idiosyncratic risk. Put differently, a risk averse investor loses more utility when his total risk (as measured by standard deviation) increases from 20% to 30% than going from 10% to 20%. Thus, for the same increase in total risk brought on by idiosyncratic risk, investors will demand a higher alpha during periods of higher systemic risk.} Yet the rise in systemic risk can have offsetting affects. If idiosyncratic risk stays fixed and systemic risk increases, the marginal increase in total risk (as measured by standard deviation) from the fixed amount of idiosyncratic risk will fall. This is easier to see in an example. Assume idiosyncratic risk is 10%, and systemic risk is 10%. In that case, total risk—measured by the standard deviation of returns—will be the square root of the idiosyncratic risk squared plus the systemic risk squared \(\sqrt{1^2 + 1^2} = 14.1\%\), or 4.1% more than the systemic risk alone. If systemic risk now rises to 30%, total risk is \(\sqrt{1^2 + 3^2} = 31.6\%\) or just 1.6% more than systemic risk alone. The decreasing contribution of idiosyncratic risk to total risk can offset the investor’s increasing sensitivity to additional risk, to the extent the investor’s utility is affected by the standard deviation (rather than the variance) of total risk. In addition, increases in systemic risk can change the shape of the distribution of returns. For example, the rise in systemic risk may cause the distribution of total returns to become asymmetric, with more returns below
Table 2 estimates the required alpha that would be necessary to compensate for bearing different levels of idiosyncratic, given different levels of systemic risk.

This would change the “skew” of the distribution. These other effects on the shape of the distribution can in theory either increase or decrease the alpha demanded when systemic risk rises.

This complex relationship is reflected in Table 2 in which, for a given level of idiosyncratic risk, alpha usually rises but sometimes falls as systemic risk rises.
### Table 2

Grid of the Additional Alpha a Risk Averse Investor Would Require to Hold Idiosyncratic Risk Under Different Conditions (% Return)

<table>
<thead>
<tr>
<th>Market Risk (Annual SD)</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>0.6%</td>
<td>1.7%</td>
<td>3.0%</td>
<td>4.8%</td>
<td>6.4%</td>
<td>8.5%</td>
<td>10.6%</td>
<td>13.1%</td>
<td>15.0%</td>
</tr>
<tr>
<td>17%</td>
<td>0.7%</td>
<td>1.6%</td>
<td>3.0%</td>
<td>4.6%</td>
<td>5.9%</td>
<td>8.6%</td>
<td>10.3%</td>
<td>12.8%</td>
<td>14.7%</td>
</tr>
<tr>
<td>19%</td>
<td>0.7%</td>
<td>1.5%</td>
<td>2.9%</td>
<td>4.5%</td>
<td>5.6%</td>
<td>8.5%</td>
<td>10.1%</td>
<td>12.7%</td>
<td>14.1%</td>
</tr>
<tr>
<td>21%</td>
<td>0.8%</td>
<td>1.7%</td>
<td>3.1%</td>
<td>4.7%</td>
<td>5.9%</td>
<td>8.3%</td>
<td>10.3%</td>
<td>12.5%</td>
<td>14.0%</td>
</tr>
<tr>
<td>23%</td>
<td>1.1%</td>
<td>2.0%</td>
<td>3.2%</td>
<td>4.3%</td>
<td>6.3%</td>
<td>7.9%</td>
<td>10.4%</td>
<td>12.2%</td>
<td>13.7%</td>
</tr>
<tr>
<td>25%</td>
<td>1.2%</td>
<td>2.1%</td>
<td>3.3%</td>
<td>3.9%</td>
<td>6.8%</td>
<td>7.9%</td>
<td>10.3%</td>
<td>12.1%</td>
<td>13.5%</td>
</tr>
<tr>
<td>27%</td>
<td>1.6%</td>
<td>2.4%</td>
<td>3.1%</td>
<td>4.5%</td>
<td>6.6%</td>
<td>8.2%</td>
<td>10.4%</td>
<td>11.8%</td>
<td>13.2%</td>
</tr>
<tr>
<td>29%</td>
<td>1.7%</td>
<td>2.3%</td>
<td>2.5%</td>
<td>5.6%</td>
<td>6.1%</td>
<td>8.3%</td>
<td>10.1%</td>
<td>11.4%</td>
<td>12.9%</td>
</tr>
<tr>
<td>31%</td>
<td>1.9%</td>
<td>1.5%</td>
<td>3.9%</td>
<td>5.3%</td>
<td>6.4%</td>
<td>8.7%</td>
<td>9.9%</td>
<td>11.3%</td>
<td>13.1%</td>
</tr>
<tr>
<td>33%</td>
<td>0.8%</td>
<td>2.9%</td>
<td>4.9%</td>
<td>5.1%</td>
<td>6.9%</td>
<td>8.8%</td>
<td>10.0%</td>
<td>11.4%</td>
<td>13.1%</td>
</tr>
<tr>
<td>35%</td>
<td>3.2%</td>
<td>4.2%</td>
<td>4.3%</td>
<td>5.3%</td>
<td>7.2%</td>
<td>8.6%</td>
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<td>13.6%</td>
</tr>
<tr>
<td>37%</td>
<td>3.7%</td>
<td>3.3%</td>
<td>4.0%</td>
<td>5.7%</td>
<td>7.2%</td>
<td>8.3%</td>
<td>9.7%</td>
<td>10.7%</td>
<td>13.5%</td>
</tr>
</tbody>
</table>

Notes: We derive these estimates of the additional alpha a risk averse investor with CRRA coefficient of 2 would require using Monte-Carlo simulations based on monthly CRSP data from 1925-2016 to create distributions with the relevant systematic and idiosyncratic risk. We can create distributions with lower risk by restricting our sample to firms with lower risk over the next year and vice-versa for higher risk. Thus, by splitting the sample in different ways, we create a distribution of annual firm returns with standard deviations ranging from 10% to 65% (i.e., we create a separate distribution with 10% total risk, 11% total risk, . . . 65% total risk). Likewise, we create different distributions of annual market returns with 15% to 37% risk and then use these distributions to estimate the additional alpha. For ease of presentation we present the relevant averages. Thus 10% idiosyncratic represents the average of 8%, 9%, 10%, 11%, and 12%, etc., and similarly 15% market risk represents the average of 15% and 16% market risk. Further details as well as a more disaggregated table are provided in the Appendix.

Table 2 helps us more clearly see why the required alpha rises during crises as both systemic and idiosyncratic risk rise, pushing southeast in the table. But the table is also useful in that it allows a more nuanced and specific assessment of how much annual alpha is required in particular circumstances. The market risk at any time can be estimated by looking at forward looking market volatility measures (such as the VIX) and the idiosyncratic risk can be similarly estimated for any stock with traded options.45 Using these two inputs, one could assess what alpha was

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45 A call (or put) option gives the owner the right to purchase (or sell) for a given price in the future. The value of these options depends in part on the expected volatility of the underlying stock from now until the option’s
necessary for more particularized situations. Thus, for example, we estimate that an Enron employee with slightly below average risk aversion (CRRA = 2) who forewent diversification to invest her retirement savings entirely in company stock would need to expect at least an average alpha of 10.8%.\footnote{To calculate this figure, we estimate an alpha for each day from January 1, 1999 to October 1, 2001 shortly before the Enron scandal finally broke and then take the average. The calculation uses the implied volatility method discussed \textit{supra} in note 45, to give forward looking measures of systemic and implied volatility. Although we do not model it here, we emphasize that investing in company stock can be particularly risky because the employees own human capital may be disproportionately exposed to idiosyncratic risk of her company. See Shlomo Benartzi, \textit{Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock}, 56 J. Fin. 1747 (2001).} In fact, we have created an online widget that lets anyone plug three variables (a level of CRRA risk aversion, a level of market risk, and a level of idiosyncratic risk for a particular stock) to determine the alpha required to take on the additional idiosyncratic risk.\footnote{\url{www.ianayres.com/alphawidget}}

Of course, not all alpha investment opportunities are quite this extreme. While the previous tables have focused on all-or-nothing tradeoffs, there are many real-world settings where investors instead of completely foregoing the benefits of diversification by investing their savings in a single stock, only partially diversify in order to reap higher expected returns. For example, some investors may invest all their savings in an actively managed fund that invests in several stocks that the fund’s managers believe will outperform the stock market generally. Investing heavily in sector funds also sacrifices some potential diversification because the investor’s portfolio bears the risk particular to that industry instead of diversifying it away by investing in the other sectors of the economy. These partially diversified positions also require offsetting alphas (but not as much as alpha opportunities that invest in a single stock).

To investigate how much alpha is required to compensate an investor for only partially diversifying, we perform another set of simulations based on historical data. In particular, in each year from 1926 to 2015, we randomly choose 1,000 representative portfolios with a given number of stocks (e.g., 20 stocks or 50 stocks).\footnote{The portfolios are representative in the sense that the probability of selecting a stock into the portfolio is proportional to the market capitalization of that firm over the course of the year.} We then calculate how much idiosyncratic risk remains in these partially diversified portfolios and using the figures underlying Table 2 convert this level of idiosyncratic risk into an alpha. We plot the results in Figure 6 for the 20 and 50 stock portfolios from 1960 to 2015:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Figure 6: Alpha required to compensate for only partially diversifying.}
\end{figure}
Each portfolio of 20 and 50 stocks for each year is created by drawing stocks randomly, without replacement, where the probability of selection is proportional to the stock’s average market capitalization over the year. The portfolio return is also market capitalization weighted (i.e., the return of large firms has a larger effect on the portfolio returns than small firms). We draw 1,000 random portfolios of 20 and 50 stocks each year. We calculate the remaining idiosyncratic risk in each portfolio and take the average across the 1,000 portfolios and then calculate an alpha associated with that average idiosyncratic risk and the level of market risk in that year for an investor with a CRRA coefficient equal to 2.

Figure 6 reveals that the required level of compensating alpha is as theory would predict substantially lower for partially diversified portfolios. While the average annual compensating alpha for a single stock over this time period is 4.77%, we find that this drops to .65% when investing in 20 stocks and to .38% when investing in 50 stocks. But importantly the picture shows that even with 50 stocks that there are 6 separate years where the required offsetting alpha is at least 1%. It is often suggested that investors can achieve the most important benefits of diversification by investing in just 10 or 20 different stocks, but our estimates show substantial variation in the requisite alpha necessary justify even relatively small departures from full diversification. During periods with relatively high systemic risk then adding even small amounts of potentially-diversifiable idiosyncratic risk can necessitate substantial alphas. If the
normal risk-premium for holding non-diversifiable market risk is 4%, then the alpha adjusted-risk-premium required for adding on just the idiosyncratic risk of a 50 stock portfolio is frequently 25% higher. The takeaway here is that even partially diversified investment opportunities can at times require relatively substantial alpha to make such an investment utility enhancing. As explained below, these results suggest that the usual rule of thumb based on these earlier studies, and still used by many fiduciaries, about how much diversification is “enough” may be too loose.

C. Exposure Costs

Finally, we estimate the “beta” costs of being non-optimally exposed to the equity risk premium. As discussed above, beta costs can come in two forms: one can have too little equity exposure (as when a 23-year old invests all her savings in money market funds), or one might have too much equity exposure (as when a risk-averse 70-year old with a modest nest egg invests all her savings in stock). And while beta costs often also require sacrificing diversification when an alpha investment is lumpy, in this section we isolate the compensating alpha required to offset having to take on inefficiently high or low beta. (In other words, we assume that there are no diversification or excessive fee losses entailed in the investment.)

As emphasized above, the notion of a beta error is only comprehensible if we have a background idea of what an optimal exposure to equities would be. That optimal level is intuitively a function of a particular investor’s level of risk aversion, which might (or might not) increase as she ages. In the following table, we assume that the optimal equity exposure is determined by the Merton share described above (as $\beta_M$) so that investors with higher constant relative-risk aversion would optimally choose to have lower exposures to equity. Using a historical distribution of returns to the market portfolio and risk premiums, we can calculate the ideal $\beta$ for an investor with any level of risk aversion. Taking investors with ideal $\beta$’s of .1, .2, …, .9, and 1 as examples, we then estimate how much alpha they would require to depart from their ideal $\beta$ in Table 3 below.

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50 5% = 4% + 1% is 25% higher than 4%.

51 For example, an investor expecting a 4.15% risk-premium, 21% equity return standard deviation and with a CRRA of 2 would optimally invest 47% of her saving in equities, while an investor with the same expectation but a CRRA of 4 would optimally invest only 23% in equities.
Table 3
Amount of Alpha Investors Would Require to Hold A Portfolio with a Beta Different than their Ideal Beta
(1 Year Holding Period)

Merton Share (i.e., Ideal Portfolio Beta)→

<table>
<thead>
<tr>
<th>Beta of Actual Investment ↓</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.3%</td>
<td>0.5%</td>
<td>0.7%</td>
<td>0.9%</td>
<td>1.1%</td>
<td>1.3%</td>
<td>1.5%</td>
<td>1.8%</td>
</tr>
<tr>
<td>20%</td>
<td>0.2%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.2%</td>
<td>0.4%</td>
<td>0.6%</td>
<td>0.8%</td>
<td>1.0%</td>
<td>1.2%</td>
<td>1.4%</td>
</tr>
<tr>
<td>30%</td>
<td>0.9%</td>
<td>0.1%</td>
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Notes: Data obtained from CRSP. The investor is assumed to have preferences which obey Constant Relative Risk Aversion (CRRA). The distribution of market returns is derived from a Monte-Carlo simulation randomly selecting (with replacement) months in the period 1926 to 2015 100,000 times and calculating the return over the following year. The geometric mean of the yearly returns for the market is 6.41%; the arithmetic mean is 8.5%. The standard deviation of the returns is 21%. The equity premium is set at 4.15%, following Ayres and Nalebuff (2013). The investor is faced with a lumpy investment opportunity, which requires her to invest her entire portfolio in it if she invests in it at all and she cannot borrow. The figures in the table represent the amount by which the investment opportunity must outperform the expected return given the investment’s beta to induce the investor to depart from her ideal share of stock in her portfolio and to invest in the lumpy opportunity instead. The relevant CRRA coefficients are as follows with the associated ideal β_M in parentheses: 0.93 (100%), 1.05 (90%), 1.19 (80%), 1.37 (70%), 1.61 (60%), 1.94 (50%), 2.44 (40%), 3.27 (30%), 4.92 (20%), 9.85 (10%). Note, these do not precisely match the figure one would get from using Merton’s formula because Merton assumes the returns are normally distributed, but empirically this is not quite true.

In Table 3, one can see the offsetting alphas from having too much or too little exposure to equities. The table shows that investors are more sensitive to beta deviations as they become more risk averse. For example, an investor with a β_M=.3 would need an annual alpha of .3% before making a beta deviation of .2, while an investor with a β_M=.7 would only need an alpha of .1% for making that sized beta deviation. More generally, the alpha required for putting risk-averse investors in high beta investments are substantially higher than the alphas required of

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For example, for an investor who ignoring the offsetting benefits of alpha opportunities would want to invest half of her savings in equities (β_M=.5), would need an offsetting annual alpha of .2% before wanting to invest her savings in a portfolio with a β of .7. Reading up and down individual columns, the table reveal substantial symmetry in the size of the offsetting alpha with regard to movements above or below the idea beta. Thus, the same investor (with β_M=.5) would also require an offsetting alpha of .2% before wanting to invest in a portfolio with a similarly sized beta deviation of .3.
relatively risk neutral investors in low beta investments. Hence, we see in the diagonal corners of Table 3, that the alpha required for putting a $\beta_M = .1$ investor into a with $\beta = 1$ portfolio is a whopping 21.4%, while the alpha required for putting a $\beta_M = 1$ investor in a $\beta = .1$ portfolio is only 1.8%. As we discuss below, this result accords with how fiduciary law has generally approached the question of beta mistakes: Not investing aggressively enough is harmful, particularly over time, but the most damaging beta mistake in the short term is exposing a highly risk-averse client—a widow who is the sole beneficiary of a small trust set up for her maintenance—to too much risk.

In most real-world contexts, the estimates in Table 3 for lumpy investments should be seen as lower bounds on the required alphas for portfolio deviations from optimal betas. This is because the lumpiness of the investments usually entail some degree of diversification loss. The opportunity to invest a substantial portion of your portfolio in a friend’s start up, for example, might force your portfolio above your optimal beta and expose your portfolio to idiosyncratic risk. Accordingly, in such circumstances it will be necessary in calculating the required alpha to account for (and offset) both types of losses. For example, if investing all of your savings in a friend’s start up caused you, a $\beta_M = .5$ ($\leftrightarrow$CRRA $\approx 2$) investor, to take on $\beta = 1$ portfolio and expose your portfolio to average non-crisis idiosyncratic and market risk, then you would need at least an alpha of 7.6%, 6.4% to compensate for the diversification loss (as shown in Table 1) and an additional alpha of 1.2% to compensate for the beta loss (as shown in Table 3).

D. Tilting Mistakes

While most of our foregoing estimates concern discrete investment opportunities, there are many real-world opportunities that give investors the option of varying the proportion of their portfolio that is invested. In such “non-lumpy” circumstances, theory suggests that an investor will want to “tilt” her portfolio toward alpha opportunities by overweighting the portfolio share of the alpha opportunity, even though this overweighting will expose the investor to some idiosyncratic risk. In this section, we investigate how much a person should invest in a non-lumpy alpha opportunity given two key variables: the size of the alpha and the total risk of alpha opportunity. As with beta mistakes, tilting mistakes can come in two varieties: an investor can undertilt by putting too small a proportion of her portfolio in the alpha opportunity or the investor can overtilt by putting too large a proportion of her portfolio in the alpha opportunity.

Table 4 estimates the optimal tilt for an investor with moderate risk aversion (CRRA = 2) depending on the size of the alpha and the riskiness of the alpha opportunity, fixing the riskiness of the market (20% standard deviation), the $\beta$ of the alpha opportunity ($\beta = 1$) and the risk premium for holding the market portfolio instead of risk-free assets (5%).

53 As discussed above in note 31, an investor’s ideal $\beta$ for a lumpy investment with idiosyncratic risk will be lower than her ideal beta for investing in a diversified portfolio. Thus, in this case, the beta error is actually worse than it first appears and the total alpha will be higher than one would calculate by adding the relevant numbers from Table 1 and Table 3.

54 Here we measure total risk as the standard deviation of investing a 100% of the portfolio in the alpha opportunity. In equilibrium, however, the total risk of the optimal portfolio will normally be reduced (toward the market risk), by investing only a portion of the portfolio in the alpha opportunity.
### Table 4

**Optimal Portion of Risky Assets Invested in "Non-Lumpy" Alpha Opportunity as a Function of Risk and Excess Return**

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Notes: The investor’s utility is assumed to obey Constant Relative Risk Aversion with a CRRA coefficient of 2. We set the risk premium at 5%, and use a β of 1 for the alpha opportunity with the market risk of 20%. The areas shaded in represent lumpy alpha opportunities an investor would choose if given the discrete choice to invest all or none of her portfolio in the alpha opportunity. Returns are assumed to be normally distributed.

Table 4 shows that when 100% tilting would expose the investor to relatively small additional idiosyncratic risk that relatively small alpha is necessary to induce an investor to want to put all of her portfolio in the alpha opportunity. For example, if going full tilt only increases the total risk standard deviation of return by 1 percentage point (from 20% to 21%), then table shows that an alpha of just 1% would be sufficient to induce an investor to want to invest all of
her portfolio in the alpha opportunity (even though she has the non-lumpy option to invest a less proportion).

But as the cost of tilting increases, investors will optimally invest a lower proportion of their portfolio in the alpha opportunity. Thus, we see if a 1% alpha opportunity has an associated total risk of 30%, an optimal investor (with CRRA risk-aversion of 2) should only want to invest 9% of her portfolio in the alpha opportunity. Under these circumstances, investing less than 9% would represent an undertilting error, while investing more than 9% of one’s portfolio would represent an overtilting error. We conjecture in real-world settings that over-tilting is the more important error. Retail investors who are not aware of the size of the gains from diversification, frequently hold only a few stocks, or even only stock in their company. Even if these investors believe the stocks will outperform the market, they are probably making a mistake by treating those stocks as though they require large minimum investments (as a share of the investor’s portfolio). Although there are situations in which the alpha is so great that one would want to hold only the alpha investment even if it is “non-lumpy”—as in the northeast corner of Table 4—these situations will be relatively rare in the real world, because they require very large alphas (or very small idiosyncratic risk).

Stepping back, we have provided in this Part some of the first estimates of the minimum alphas that are required to offset diversification, beta losses, as well as excessive fees. But for a variety of reasons these estimates should be viewed as ball-park measures. For example, our estimates on diversification losses assume that investors have a particular form of (“constant relative”) risk aversion. Other types of risk aversion are less mathematically tractable but might be more empirically relevant and give rise to alternative estimates. In addition, we have not modeled investor’s exposure to the market through their human capital. Also our estimates have assumed that investors “know” a variety of variables, including the alpha of particular investment opportunities and the level of idiosyncratic and market risk. But in many situations investors are likely to have varying degrees of confidence in their beliefs about alpha and these other variables. While the expected market volatility is derivable from options prices, investors’ beliefs about idiosyncratic risk and alphas might be less precise. For risk averse

55 The importance of diversification, fees, and appropriate exposure to market risk has long been understood, but we are unaware of other work that has systematically converted the losses from sacrificing diversification or beta mistakes into an alpha required to justify them. Doing so is informative because most work simply treats failure to fully diversify or expose one’s portfolio to the right level of market risk as an important mistake, but does not tell us when these costs are worth bearing.

56 Scholars have also at times analyzed constant absolute risk aversion (CARA) and hyperbolic absolute risk aversion (HARA). See Daniele Schilirò, Bounded Rationality And Perfect Rationality: Psychology Into Economics, 3 THEOR. & PRACTICAL RESEARCH IN ECON. C FIELDS 101 (2012). But see supra, note 18 (suggesting that constant relative risk aversion is a reasonable way to approximate investor’s risk aversion).

57 See supra notes 43 and 46.

individuals, less precise beliefs about alpha should militate toward demanding even higher alphas because uncertainty about alpha is another form of risk.\textsuperscript{59}

Even with these caveats, the take-home result of this section is that investors need to have reasonable expectations that an investment will substantially beat the market before being willing to take on diversification, beta, and excess fees losses. Financial economists normally expect that stocks will beat government treasuries by somewhere between 3 and 6 percentage points. But an investor who puts all her savings in a single stock would need an additional alpha of at least twice this amount (6.4\% in Table 1) and during crisis periods an alpha of nearly 10\% annually. Moreover, if the opportunity requires the investor to pay excessive fees, the alpha should be calculated net of this excess and if the investment necessitated a beta deviation, an additional alpha to offset the exposure loss would be required. Investment opportunities with alphas of these magnitudes are not impossible, but are likely to be sufficiently rare that the law should be quite concerned when fiduciaries advise clients to take on substantial diversification, beta, or excess fees costs, or in the case of trustees, directly invest the beneficiaries’ funds in that manner.

IV. LEGAL IMPLICATIONS

The last two Parts analyzed the theoretical and empirical tradeoffs that often arise when investors pursue alpha investment opportunities. This Part develops the legal implications of these tradeoffs. More particularly, we describe what we call “alpha duties,” the legal duties that investment fiduciaries should have before recommending alpha investments or investing in such opportunities on their clients’ behalf. The part is organized around three types of fiduciaries: (a) trustees, (b) broker-dealers and investor advisors, and (c) 401(k) and other ERISA fiduciaries. The next section on trustees lays out the core limitations concerning recommendation and actual investment in alpha opportunities, while the subsequent sections explore specialized questions regarding upgraded licensing requirements of broker dealers and investment advisors as well as “alpha-tized” 401(k) menu selections and fintech warnings.

A. Trustees

A trust separates legal and beneficial ownership of property. The trustee is by default given all the powers over the property of a sole owner to manage and invest it for the advantage of the beneficiaries.\textsuperscript{60} Thus, the trustee doesn’t just recommend (as is often the case with investment advisors and broker dealers), instead the trustee makes the ultimate decision about how to invest the property in question. Under these circumstances, stronger fiduciary protection is likely to be needed than in the contexts of mere recommendation (where the client is empowered to waive off the recommendation).\textsuperscript{61} It should also be noted that these duties are

\textsuperscript{59} See, supra, note 41.

\textsuperscript{60} Uniform Trust Code (hereinafter “UTC”) §815; see also Uniform Prudent Investor Act §2(e).

\textsuperscript{61} Fiduciary duties are not the only possible disciplining forces for trustees. Market forces provide some check on commercial trustees who can garner new clients by showing superior investment returns. See Jeffrey N. Gordon, \textit{The Puzzling Persistence of the Constrained Prudent Man Rule}, 62 N.Y.U. L. REV. 52, at 84 (1987). This seems likely to be insufficient on its own given the difficulty of separating skill and luck, particularly by trust settlors and others who are usually not investment professionals.
mostly imposed by default law—i.e., the person creating the trust (the “settlor”) can usually opt out if she chooses—but the default is nevertheless highly influential.\footnote{There is a robust debate, however, about whether the duty to diversify is simply a default which can be waived for nearly any reason or is mandatory unless the settlor has a good reason to waive it. See John Langbein, \textit{Mandatory Rules in The Law of Trusts}, 98 NW. U.L. REV. 1105 (2004). One commentator summarized this piece, writing: In a 2004 article, Professor John Langbein examined the UTC’s mandatory rules. He argued that the duty to diversify investments cannot be entirely waived by the settlor. Rather, just as a settlor cannot create a trust for capricious purposes, so, too, a settlor should not be permitted to waive the duty if it would violate the benefit-of-the-beneficiaries rule. In other words, the settlor’s investment-related restrictions should not be respected if it would impair the value of the portfolio and thereby inure to the detriment of the beneficiaries. In his example involving IBM stock, Professor Langbein posited a case where the trust instrument directed the trustee not to diversify. He explained that modern portfolio theory has shown that such non-diversification creates a risk for which the investor is not compensated and that the settlor should not be permitted to impose foolishly this harm on the beneficiaries. Mitchell Gans, \textit{Duty to Diversify: Default v. Mandatory Law}, https://trustest.jotwell.com/duty-to-diversify-default-v-mandatory-law/. But see Jeffrey A. Cooper, \textit{Empty Promises: Settlor’s Intent, The Uniform Trust Code, and the Future of Trust Investment Law}, 88 B.U. L. REV. 1165 (2008) (arguing the requirement to benefit the beneficiaries standard does not impede terms, including mandating an undiversified portfolio, that are likely foolish but not illegal, immoral, nor against public policy).}

Trust law has long contained a stringent duty of loyalty which prevents trustees from benefiting from personal transactions involving trust property.\footnote{See Max M. Schanzenbach & Robert H. Sitkoff, \textit{Did Reform of Prudent Trust Investment Laws Change Trust Portfolio Allocation?}, 50 J. LAW \& ECON 681 (2007) (finding that after adopting new prudent-investor rule, institutional trustees held about 1.5-4.5 percentage points more stock at the cost of “safe” investments.)} Providing robust rules to ensure the trustee invests prudently, however, has proved harder. Prior to the 1980s, most states limited the types of property a trustee could invest in either through formal lists or the “constrained prudent man rule.” Both doctrines channeled trust property into bonds or real property, and banned investment in some or all equities.\footnote{See, e.g., In re Gleeson, 124 N.E. 2d 624 (Ill. App. 1954) (holding the trustee liable under the ‘no further inquiry rule’ where the trustee benefits from a transaction with trust property, even though the transaction was very likely in the best interest of the trust).} The rules frequently prevented trustees from giving beneficiaries enough exposure to equities and during the high inflation periods in the 1970s and 1980s, bond-heavy trust portfolios floundered. Observing these failures, trust law reformers succeeded during the 1990s in breaking down the previous constrained approach. The Uniform Prudent Investor Act (“UPIA”) (1994), eventually adopted in 45 states, abrogated bans on investing in categories of risky assets and instead requires the trustee simply to “manage trust assets as a prudent investor would, by considering the purposes . . . [and] circumstances of the trust.”\footnote{John Langbein, \textit{The Uniform Prudent Investor Act and the Future of Trust Investing}, 644-645.} The remaining states adopted similar measures, although not based on the UPIA language.

Although this prudent investor standard provides useful flexibility, some commentators have complained that it fails to prevent trustees from making what we’ve called “beta” or...
exposure mistakes by investing in portfolios which are too risky for the beneficiaries. Likewise, while the UPIA imposes a default duty to diversify, it provides relatively little guidance as to how much diversification is enough or when circumstances make a relatively undiversified portfolio prudent. Our results can help to address these problems by fleshing out the meaning of prudent investing and the duty to diversify under the Act.

Specifically, our analysis suggests that the fiduciary duty of prudence should be enhanced with both procedural and substantive safeguards whenever a trust portfolio fails to diversify, takes on risk that is excessive (or insufficient) relative to the beneficiaries’ level of risk-aversion, or pays excessive fees. Whenever a trust investment decision is justified by the trustee’s belief that the investment or set of investments will beat the market, the trustee should have procedural evidence that the trustee considered the types of alpha tradeoffs analyzed above. Moreover, the trustee should explicitly estimate the costs of excessive fees, failing to diversify, and/or deviating from otherwise would be optimal exposure. The trustee should separately estimate and justify the expected alpha from the investment decision, and show that the alpha benefits exceed the associated costs.

These alpha duties can be easily incorporated into the trustee’s existing procedural duty to keep adequate records, which includes recording the reason for major investment decisions. Most professional trustees already have investment protocols in place which document the trust’s purposes, the needs of the beneficiaries, and reasons for investment decisions, including maintaining relatively concentrated positions (over 10 or 20%) in certain securities. Courts often find the failure to set up an investment plan in a timely manner or adhere to an internal protocol as probative of whether the trustee has acted imprudently.

Our analysis also suggests that these justificatory duties should be ongoing—especially with regard to failures to fully diversify. The trustee should have in place mechanisms to diversify the trust if total risk from the portfolio becomes too high. This dynamic mechanism will often include a VIX contingent trigger which would presumptively trigger a rebalancing of an alpha portfolio toward a more diversified set of investments if market risk became too high. Trustees should also monitor over time and across clients how often their alpha investments actually beat the market. While alpha opportunities are not expected to uniformly produce returns above the market’s return, keeping track of return of outcomes compared to the trustee’s

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68 See UTC § 810(a); JESSE DUKEMINIER & ROBERT H. SITKOFF, TRUSTS, WILLS AND ESTATES 385, 633-34 (2013).
69 See DUKEMINIER & SITKOFF, supra note 68, at 633. As discussed below, these cutoffs for possible under-diversification are probably too loose, at least during periods of market upheaval.
70 See In re Estate of Janes, 681 N.E. 332 (N.Y. 1997); In re Hunter, 995 N.Y.S.2d 163 (App. Div. 2012). As one commentator noted “[t]rustees who understand both the power of hindsight bias and the weakness of uncorroborated testimony will document every potentially important decision clearly, completely and contemporaneously.” Randall W. Roth, Hindsight Bias and the Curse of Knowledge: Forewarned Is Forearmed, ABA Tr. & Inv., Jan./Feb. 2011, at 33.
71 See Alan Moreira & Tyler Muir, "Volatility Managed Portfolios" (working paper 2016) (“Managed portfolios that take less risk when volatility is high produce large alphas”).
ex ante alpha assessments can provide valuable feedback that can allow trustees (as well as settlors, beneficiaries and courts) to update their beliefs about the trustee’s ability to identify true alpha opportunities.\textsuperscript{72}

Our results also militate toward the possible creation of a new duty (or at least best practice) on the part of the drafter of the trust to warn settlors who are interested in setting up a trust which would depart from the diversified, low-fee, appropriate risk benchmark about the offsetting alpha that would be required to justify that departure.\textsuperscript{73} Courts might even establish a cautionary “altering rule,” mandating that to be effective, a trust instrument opting out of the default duty to diversify must indicate that the settlor has been apprised of and understands the alpha tradeoff relevant to the investment restriction that the settlor wishes to put in place.

We should caution, however, that our estimates would need to be adjusted to the facts and circumstances of a trust before they could be used to assess the prudence of a given investment decision. Nevertheless, our estimates for the alpha required by investors to accept (1) excessive fees, (2) a failure to diversify, or (3) more risk than they would ideally prefer, provide useful benchmarks that can be used as rules of thumb for courts and trustees.

\textit{Diversification.} Under UPIA, the trustee has a duty to diversify the trust portfolio “unless the trustee reasonably determines that, because of special circumstances, the purposes of the trust are better served without diversifying.”\textsuperscript{74} The official comment lists two common circumstances in which not diversifying might be prudent: (1) if the trust consists in part of property with a low tax basis or (2) if the trust contains a family business. In addition, the UPIA permits prudent active management of trust assets,\textsuperscript{75} which entails not fully diversifying. The official comments to the UPIA note that “[t]here is no automatic rule for identifying how much diversification is enough” and cite the 1992 Restatement’s advice that “[s]ignificant diversification advantages can be achieved with a small number of well-selected securities

\textsuperscript{72} The value of updating decision maker beliefs based on interim outcomes is central to many areas of evidence based policy making. See IAN AYRES, SUPER CRUNCHERS: WHY THINKING-BY-NUMBERS IS THE NEW WAY TO BE SMART 113 (2007) (arguing that parole boards should particularly pay attention to recidivism rates when board overrides algorithm and paroles prisoners deemed by board to be low risk).

\textsuperscript{73} An interesting separate question is to what degree a settlor can impose her level of risk-aversion on a trust intended to benefit another. One might easily conjure examples with regard to a donative trust in which the settlor wants the trust to take less risk than the beneficiary would like (or more risk than the beneficiary would like). We would propose that the trust drafter warn a settlor who expressed interest in waiving the duty to diversify or in imposing, given the circumstances, a relatively risky (or overly cautious) investment strategy about the alpha tradeoffs from the point of view of each of the beneficiaries, as well as of the settlor—so that the settlor has fuller information about the impact of the deviation on the welfare of all concerned. Too wide a divergence between the needs of the beneficiary and the settlor’s chosen investment strategy—e.g., a trust invested solely in a penny-stock for the benefit of an otherwise penniless widower—would violate the mandatory rule that the trust’s purpose must be to benefit the beneficiary. See, Lanbein, supra note 62.

\textsuperscript{74} See Max M. Schanzenbach & Robert H. Sitkoff, \textit{Did Reform of Prudent Trust Investment Laws Change Trust Portfolio Allocation?}, 50 J. LAW & ECON 681, 708-709 (2007) (reporting enactment of the UPIA by 42 states and noting that the other states enacted laws similar to the UPIA).

\textsuperscript{75} See Sterk, \textit{supra} note 67, at 896 (2009) (citing UPIA § 1 (a) (1994) and the Restatement 3\textsuperscript{rd} of Trusts as authorizing active management)
representing different industries . . . . Broader diversification is usually to be preferred in trust investing.”

Although the UPIA does not set up a formal shifting of the burden of persuasion or going forward, the text of the rule makes it incumbent on the trustee to make a showing that her decision was reasonable, if she fails to diversify. But that begs the question: how much concentration of the trust portfolio must a complaining beneficiary show before the trustee owes an explanation? The leading textbook in Trusts and Estates notes the received wisdom: “In light of the studies showing that diversifying into 20 to 30 unrelated large capitalization stocks removes nearly all of the diversifiable risk from a stock portfolio, a common rule of thumb is that a concentration in a single security of more than 5 percent requires explanation.” Our work shows that during volatile periods when idiosyncratic risk is high, this rule of thumb is too loose. A random, market-weighted, portfolio of 50 stocks will have few or no stocks with concentrations above 5%, but will still impose high costs during volatile periods. These under-diversification costs have been up to 150 basis points per year for moderately risk averse beneficiaries during unsettled periods. By comparison, during calm periods like 2003-2007 and 2011-2015, a random market-weighted portfolio of 10 stocks would have imposed on much smaller—although certainly non-trivial—average costs of about 60 basis points for the same investor. This is true despite the 10-stock portfolio raising serious red flags under the 5% concentration rule of thumb, while the 50-stock portfolio would not. During volatile periods, concentrations of the trust corpus in individual stocks of more than two or three percentage points over the firm’s share of the market as a whole should usually be considered not diversified, triggering an explanation from the trustee.

To give a concrete sense of how our work can provide guidance for courts evaluating whether trustees have fulfilled their fiduciary obligations consider a private trust that has invested all of the trust’s assets in a single stock. The trustee justifies the failure to diversify because selling the stock would trigger a capital gains tax realization. In this case, the alpha the trustee hopes to obtain is from the tax savings and is readily calculable. For simplicity, imagine the stock has a 0 tax basis, average idiosyncratic risk, and a single beneficiary with moderate risk.

76 See, e.g., In re HSBC Bank USA, NA, 947 N.Y.S.2d 292 (App. Div. 2012) (“[T]he prudent investor rule puts diversification at the forefront of the fiduciary's obligations, but allows leeway for the fiduciary to opt out if the beneficiaries require otherwise.”).
78 See, supra Figure 6 (additional alpha required to hold random 50 stock portfolio for CRRA = 2 investor instead of market index).
79 Tightening the rule of thumb has real costs in potentially increasing the number of fiduciary suits and forcing the trustee to spend more time recording her reasons for holding a concentrated position. Still, rearranging the rule of thumb to slide depending on whether it is a relatively calm or relatively volatile period could improve the trustee’s incentives without raising total costs.
80 The same is true if the trustee indirectly holds an equivalent position through actively managed mutual funds or some other mechanism.
81 Note that the rule of thumb works only one way. A portfolio that passes the “test” is not necessarily diversified. For example, a portfolio holding only U.S. stocks with no exposure to other risky assets like real estate, international equities, or bonds may not be appropriately diversified.
aversion (with a CRRA coefficient of 2) during a relatively calm period. We calculated (in Table 1, supra) the benefit to diversifying would be the equivalent to adding 2.9%-6.3% to the return to the stock for the year. Ignoring the step-up basis at death,\textsuperscript{82} triggering the tax this year rather than postponing it would cost the trust:

$$a = \left( \tau \times (Gains) \times r_f \right) \times (1 - \tau),$$

where $\tau$ is the tax rate, and $r_f$ is the risk free rate. The first part of this equation $\left( \tau \times (Gains) \times r_f \right)$ represents the lost time value of money in failing to delay recognizing the untaxed gains, while the second part $(1-\tau)$ represents the fact that investors only capture the after-tax portion of these gains. This expression represents the excess, alpha-like, return the trust can expect from postponing realization. Even taking the risk-free rate as high as 9%, and if $\tau = 20\%$ is the long-term capital gains rate, then the tax benefit to not diversifying is 1.4% of the value of the untaxed gain and it would be imprudent not to diversify. The alpha benefit from non-diversify (1.4%) is outweighed by the cost of not-diversifying (2.9-6.3%).

Under other circumstances, the tax benefits would justify remaining undiversified. For example, if the trust instead held a portfolio of 10 random stocks with 0 basis, whether it would be prudent to fully diversify might depend on the volatility of the market. During volatile periods the benefits of diversification increase, but the tax benefits are roughly fixed. Thus, during calm periods when the diversification benefits are only around .5% it might be prudent to not fully diversify, thus saving the tax costs of 1.4%. During more volatile times, however, it would be imprudent not to diversify. For example, the average benefit to diversifying during 1999-2001 or 2008-2009 was 2.2% for a beneficiary with CRRA coefficient equal to 2, well in excess of the (1.4%) tax costs.\textsuperscript{83}

Under other circumstances the gains from concentrating the trust portfolio may be harder to calculate, but it can still be quite important to estimate the costs of having a concentrated portfolio. For example, trusts are often used to perpetuate family businesses. In these businesses, the benefits of not diversifying—e.g., employment at the firm for family members, perquisites, family sentiment and pride of ownership, etc.—are diffuse and difficult to quantify. Still the trustee can estimate the cost of failing to diversify — that is, the offsetting alpha required to make concentrating the trust assets in the family firm prudent — by using the average of small publicly-traded firms in the same sector.\textsuperscript{84} This provides a benchmark against which the

\textsuperscript{82} Under I.R.C. §1014(a), a person taking most kinds of property from a decedent receives tax-basis equal to the current fair-market value, regardless of the decedent’s original basis. The decedent’s final income tax return likewise does not include gains (or losses) built into property held at death. For property with built-in-gains at the time of the decedent’s death, the beneficiary’s basis is thus “stepped up” from the original basis to the current fair market value, and the result is that the “accrued gain . . . gain on the property will never be subject to income tax.” \textsc{Michael Graetz & Deborah Schenk}, \textit{Federal Income Taxation: Principles and Policies} 148 (6th ed. 2009). Many trust arrangements, however, particularly those which are irrevocable by the settlor and in which she retains no interest, will not entitle the trust to such a step-up basis at the settlor’s death.

\textsuperscript{83} Even during the calm periods, it would likely be prudent to sell off part of the low basis portfolio and diversify that portion.

\textsuperscript{84} Thus, the same method applied to all stocks (reported in the CRSP database) to produce the alpha estimates of 2.9% to 6.3% in Table 1 could be applied just to publicly traded firms in the same SIC (Standard Industrial
benefits of control can be weighed. Moreover, these costs of not being diversified are likely to change significantly over time, with costs rising during volatile periods. Thus, all else equal, a prudent trustee will be more likely to seek or accede to a bid for the family firm in the midst of an unsettled market, than in calm periods. This is true even if the mean expected return for the firm was the same in both periods.

Frequently, the trustee is not only permitted to retain the concentrated position in a family firm, but required to do so by the settlor in the trust instrument. In such cases, the firm’s prospects or level of idiosyncratic risk may change in ways not anticipated by the settlor. Increases in risk may force the trustee to petition the court to allow her to sell the firm to avoid serious harm to the beneficiaries. This is known as an equitable deviation and serves the settlor’s ultimate purpose to benefit the beneficiaries, even at the cost of overruling the settlor’s preference for retaining the family firm. Our results demonstrate that courts should be most amenable to these petitions during periods when idiosyncratic risk is high.

The rule that courts should be more sympathetic to petitions to allow sale of concentrated assets during volatile periods is consistent with the most famous equitable deviation case, In re Pulitzer. The case arose from the potential sale of the New York World newspaper during the throes of the Great Depression. In 1931, the trustees of Joseph Pulitzer’s testamentary trust petitioned the court to allow the sale of the New York World, which trust was required to hold. At that time, the paper was foundering and the early years of the Great Depression were a period of enormous market upheaval and spikes in idiosyncratic risk. A moderately risk averse person would have needed to receive a staggering 162% expected alpha to make him willing to hold one of the publicly traded newspapers during the period, instead of a diversified portfolio of U.S. stocks, as compared with 19% from 1926-September 1929. Although the court did not use the language of risk, it wisely concluded that changed circumstances unforeseen by Pulitzer required the trustees to be allowed to sell the paper.

Classification) or NAICS (North American Industry Classification System) industry code. For example, when we do this for the newspaper and periodical field we estimate an offsetting required alpha of 4.9% for investors with moderate risk aversion (CRRA=2).

85 The trustee has a duty to petition the court if she knows or should know of changed circumstances which have “the potential to cause substantial harm to the trust or its beneficiaries” and which can be avoided by changing the terms of the trust. See RESTATEMENT (THIRD) OF TRUSTS § 66(l-II) (2003).

86 Although most trust law is default law which is waivable by the settlor (including explicitly the UPIA), the settlor’s overriding purpose must be to benefit the beneficiaries. That rule is mandatory. See Langbein, supra note 66, at 391 (arguing the mandatory purpose to benefit the beneficiaries requires the courts to strike foolish conditions on investment of the corpus by the settlor). But see Jeffrey A. Cooper, Empty Promises: Settlor’s Intent, The Uniform Trust Code, and the Future of Trust Investment Law, 88 B.U. L. REV. 1165 (2008) (arguing the requirement to benefit the beneficiaries standard does not impede terms that are likely foolish but not illegal, immoral, nor against public policy).

87 In re Pulitzer, 249 N.Y.S. 87 (Sur. 1931).

88 There were six newspaper companies traded on the NYSE or American Stock Exchange during the relevant period. The average (not weighted by market cap) idiosyncratic risk for these firms during 1931 was 6.2% per day. The alpha calculation is made using the normal distribution—due to the fact that there are not enough firms to use a Monte Carlo simulation—for an investor with an CRRA coefficient equal to 2 and assuming the worst possible outcome is for the investor to lose 99.9% of her investment.
Prudent Exposure to Risky Assets

The drafters of the UPIA recognized that while no assets are categorically imprudent for all beneficiaries, creating a portfolio with high systemic risk is usually imprudent for trusts meant for highly risk averse beneficiaries, i.e., “widows and orphans” trusts. Part III.C above quantifies this intuition, showing that, unsurprisingly, that the trustee should expect very large alphas before it would be prudent to invest the portfolio of highly risk averse beneficiaries entirely in the market. A risk-averse investor who but-for the alpha opportunity would only rationally invest 10% of her portfolio in equities would need an offsetting annual alpha of 21.4% before investing in a diversified all-equity portfolio with a beta of 1. Opportunities with so much alpha are, of course, rare. By contrast, relatively risk-tolerant individuals (CRRA = 1) would happily take very low exposure to risky assets if they could obtain alphas one tenth that size.

Our calculations can serve as benchmarks to help courts understand whether a trustee has taken too much risk in the portfolio. Some scholars have argued that by eliminating the legal list and constrained prudent man approach to trust investing, the UPIA eliminated the main checks on trustees’ incentives to take too much risk without imposing an effective replacement. They argue trust settlors choose trustees in part based on the trustees’ past returns, but will not fully account for the risks taken, leading to trustees taking excess risk. Moreover, the UPIA’s prudence standard gives little substantive guidance about what the appropriate level of risk actually is. Our results cannot give an exact answer to the appropriate level of risk, but confirm the intuition that it is likely to be imprudent to invest in portfolios with β of 0.5 or more for highly risk averse beneficiaries, even when presented with alpha opportunities as large as 360 basis points. By contrast, the trustee of “a trust to accumulate for a young scion of great wealth” (who is presumably relatively risk tolerant) can reasonably choose to invest in less risky assets with a β that is lower than ideal, say of 0.5, if she believes there is an opportunity to gain alpha as little as 50 basis points.

Excessive exposure to market risk becomes more costly during crisis periods, because even with full diversification, the systemic risk of equities tends to be higher during crisis periods. Hence the cost of taking on inefficiently high market exposure is particularly costly. For example, during the most recent financial crisis, market risk more than doubled, leading to the ideal Merton share for investors falling by half (if the risk premium is assumed to have been unchanged). This means that a trustee who was taking on only a bit too much risk in the period before the crisis, would have been making a much larger mistake during the crisis if she did not scale back the trust’s exposure to stocks.

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89 See supra Table 3.
90 See Sterk, supra note 67, at 882, 887.
91 See id. at 885-889.
92 UPIA §2 cmt. ("Risk and return").
In addition, in both the low tax basis and family firm scenarios discussed above, a prudent trustee must consider not only the costs of the failure to diversify, but also the potential mismatch between the ideal exposure to risky assets for the beneficiaries and the exposure provided by the concentrated portfolio. For concentrated portfolios with relatively low exposure to risky assets, this can lead to counterintuitive results. A prudent trustee given a low beta (e.g. \( \beta = .5 \)), low tax basis portfolio of 50 stocks might conclude that she should sell the portfolio if the beneficiaries are highly risk averse or only slightly so, but should hold it if the beneficiary is in the middle in terms of risk aversion. For the highly risk averse, the diversification benefits outweigh the tax costs, and for the relatively risk tolerant beneficiaries the costs of having too little exposure to the market may exceed the tax costs. For moderately risk averse beneficiaries, however, the portfolio provides close to the optimal beta and the diversification benefits are smaller than for more risk averse beneficiaries and can thus be less than the tax costs.

B. FINRA Fiduciaries (Broker Dealers and Registered Investment Advisors)

Broker Dealers and Registered Investment Advisors are required by a FINRA (Financial Industry Regulatory Authority) Rule to only recommend “suitable” transactions:

A member or an associated person must have a reasonable basis to believe that a recommended transaction or investment strategy involving a security or securities is suitable for the customer, based on the information obtained through the reasonable diligence of the member or associated person to ascertain the customer's investment profile. The rule requires firms and associated persons to seek to obtain information about “the customer's age, other investments, financial situation and needs, tax status, investment objectives, investment experience, investment time horizon, liquidity needs, [and] risk tolerance.”

The rule requires firms and associated persons to seek to obtain information about “the customer's age, other investments, financial situation and needs, tax status, investment objectives, investment experience, investment time horizon, liquidity needs, [and] risk tolerance.” FINRA as a self-regulatory organization may discipline or remove members for violating the suitability rule, but it does not by itself clearly give rise to a private right of action.

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93 When there are multiple beneficiaries with different risk preferences the problem of finding the optimal exposure to risky assets is complex and is not usually optimized by the “Solomonic” portfolio which is midway between the ideal portfolio of each of the beneficiaries. See James R. Hines, Jr., Efficient and Impartial Trust Investing (working paper) (manuscript on file with authors).


96 See Macey et al., supra note 94 at 817 n. 138, citing Jablon v. Dean Witter & Co., 614 F.2d 677 (9th Cir. 1980) (which found no private right of action), and Thompson v. Smith Barney, Harris Upham & Co, 539 F. Supp. 859 (D. Ga. 1982), aff'd, 709 F.2d 1413 (11 th Cir. 1983) (rejecting the plaintiffs’ theory that NASD and NYSE suitability rules create a private right of action). Not all courts have agreed with this line of cases, see Colonial Realty Corp. v. Bache & Co., 358 F.2d 178 (2d Cir. 1966), which found that, under certain circumstances, there can be private actions under SRO rules. Nevertheless, “since Touche Ross & Co. v. Redington, 442 U.S. 560 (1979) and Transamerica Mortgage Advisors, Inc. v. Lewis, 444 U.S. 11 (1979), the trend of courts is overwhelmingly against
Courts, however, have allowed found broker dealers and investment advisors liable under Rule 10b-5 “for selling an unsuitable security is that when recommending an inappropriate security to a customer the defendant either provided misleading information or omitted to state a material fact to the investor.” 97 To recover in a Rule 10b-5 action:

A plaintiff must prove (1) that the securities purchased were unsuited to the buyer's needs; (2) that the defendant knew or reasonably believed the securities were unsuited to the buyer's needs; (3) that the defendant recommended or purchased the unsuitable securities for the buyer anyway; (4) that, with scienter, the defendant made material misrepresentations (or, owing a duty to the buyer, failed to disclose material information) relating to the suitability of the securities; and (5) that the buyer justifiably relied to its detriment on the defendant's fraudulent conduct. 98

To be held liable, an investment fiduciary must have scienter, but some courts have interpreted each of these elements broadly – for example, by finding that mere "recklessness" by brokers is sufficient. 99

The suitability duties can easily be adapted to respond to analysis of this paper. The central concern with trustees was whether they had under taken sufficient procedural and substantive precautions before investing a trust corpus in ways that tradeoff expected alpha against the costs of diversification, beta or excess fees losses. But with respect to broker dealers and investment advisors, the analogous concern is whether the recommended investment passes the same cost-benefit analysis. Accordingly, a recommendation that exposes a client’s portfolio to diversification, beta or excess fee losses should only be deemed as suitable if the fiduciary expects the recommended investment to produce a sufficiently high alpha. As with trustees, the fiduciary recommenders should (i) substantively estimate the costs of excessive fees, failing to diversify, deviating from otherwise would be optimal exposure, (ii) separately estimate and justify the expected alpha from the investment decision, and (iii) show that the expected alpha benefits exceed the alpha-required to offset these associated costs. And as before, the broker dealers and investment advisors should have mechanisms in place to update their

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97 Macey et al., supra note 94, at 818; Poser, supra note 96, at 1537-1554.
98 Brown v. E.F. Hutton Group, Inc., 991 F.2d 1020 (2d Cir. 1993); see also Platsis v. E.F. Hutton & Co., 946 F.2d 38, 40 (6th Cir. 1991)
99 See O'Connor v. R.F. Lafferty & Co., 965 F.2d 893, 899 (10th Cir. 1992) ("Therefore, in our test for unsuitability a plaintiff must show the broker purchased the securities with an intent to defraud or with reckless disregard for the investor's interests."). But see NORMAN S. POSER, BROKER DEALER LAW AND REGULATION, § 3.03 (noting that it is unsettled whether recklessness suffices).
recommendations based on evolving market conditions and to keep track of their success across clients with regard to predicting alpha.\(^\text{100}\)

As with trustees, broker dealers have been criticized for beta mistakes—with courts finding recommendations unsuitable that expose clients to risks that are excessive given the client’s level of risk aversion.\(^\text{101}\) But courts have not been sufficiently attentive to what level of alpha could justify beta deviations. Accordingly, suitability inquiries informed by our analysis can lead to more nuanced imposition of liability that simultaneously corrects type I and type II errors of current jurisprudence.

Suitability duties can also be deployed to induce fiduciary resistance to unsuitable trades that are prompted or directed by the client. While the suitability rule gives more sophisticated clients greater leeway in self-directing their accounts,\(^\text{102}\) broker-dealers “cannot recommend, and subsequently help customers engage in, unsuitable transactions even if all risks are disclosed to the customer and the customer still requests to become involved in the transaction”:\(^\text{103}\) The SEC has stated that “[a]s a fiduciary, a broker may only make recommendations that are in the best interests of his customer, even when the recommendations contradict the customer’s wishes.”\(^\text{104}\) Under the suitability rule, a broker-dealer cannot recommend a transaction that is appropriate for a client’s stated goal if the goal is not suitable for the client’s financial situation. Thus, even if a client asks for a high rate of return, a broker cannot recommend a set of high risk/reward securities, if that portfolio would not be suited to the customer’s financial situation.\(^\text{105}\)

A suitability requirement, however, is only triggered if there is a fiduciary recommendation. Hence, a client who instructs a broker dealer to buy particular securities imposes no resistance duty—such as a duty to inquire whether the client expects a sufficient alpha to offset the likely diversification, fee or exposure losses of the investment. But it would certainly be best practices for fiduciaries to proactively (i) inquire whether client-directed investments are supported by a client’s expectation of sufficient alpha, when an investment

\(^{100}\) Excessive churning of portfolio investments is a species of excess fees that should be alpha justified. Under FINRA “quantitative suitability” regulations, broker/dealers are required “to have a reasonable basis for believing that a series of recommended transactions, even if suitable when viewed in isolation, are not excessive and unsuitable for the customer when taken together in light of the customer's investment profile, as delineated in Rule 2111(a).” FINRA Rule 2111.05(b) and (c). The regulation’s list of factors defining excessive trading activity should include alpha evidence related to the possibility that purchases or sales will yield excess returns. FINRA also regulates excessive fees by prohibiting “breakpoint sale” transactions when “a registered rep fails to inform, or remind a mutual fund investor of the availability of reduced sales charges at breakpoints, in particular when that investor is purchasing an amount not far from a breakpoint.” Id.

\(^{101}\) See Macey et al., supra note 94; Hills, Oxley & Holinsky, supra note 94, § 6:1.2, at 6-4.

\(^{102}\) See Macey et al., supra note 94.

\(^{103}\) See id. at 815.


\(^{105}\) See Macey et al., supra note 94; Pinchas, Exchange Act Release No. 41,816, 70 SEC Docket 1108 (Sept. 1, 1999), 1999 WL 680044, at *6. In Pinchas, the SEC upheld sanctions imposed by the NASD (a predecessor to FINRA) against a broker, Rafael Pinchas, finding that “even if [the client] had desired Pinchas to double her money, that desire would not have relieved Pinchas from his duty to recommend only those trades suitable to her situation.”
exposes to diversification or other costs described above, and (ii) recommend alternatives. And as discussed below, in a world with fintech algorithms, such inquiries/warnings/recommendations could be automatically and cheaply provided.

In addition, to the suitability obligations that arise at the time of fiduciary recommendations, the tests required for licensing should be enhanced to assure that fiduciaries are cognizant of the three tradeoffs at the heart of our analysis. FINRA requires both broker dealers and registered investment advisors to pass exams that include sections covering the suitability requirement.107

But the questions on these exams fail to test applicants on whether failures to diversify or take appropriate levels of risk or to minimize investment fees can be justified by expectations of excess returns.108 Enhanced testing should assure that FINRA fiduciaries have both a theoretical and empirical understanding about the central alpha tradeoffs. For example, they should not only know theoretically that some alpha is required before sacrificing the benefits of diversification (and that it tends to increase during crisis periods), but they should also know empirically what order of magnitude this alpha must be for clients of different levels of risk aversion. They should be tested on what alpha is required before taking on too much or too little risk for their clients. And most simply, they should know that any superficially excessive fees on a mutual fund must be alpha-justified, by even higher alpha expectations. Assuring ex ante that licensed fiduciaries have this kind of knowledge will better position them to follow through on the enhanced alphatized suitability requirements laid out above and hence make more suitable investment recommendations for their clients.

C. 401(k) and Other ERISA Fiduciaries

Finally, we take on the implications of our analysis for 401(k) fiduciaries and other ERISA fiduciaries.109 Under the Employee Retirement Income Security Act of 1974 (ERISA), sponsors of 401(k) retirement plans who manage the plan or exercise discretionary authority over the plan’s assets are fiduciaries and are required to exercise control solely in the interest of plan

106 See infra text accompanying note 113.

107 Before recommending transactions involving stocks, bonds and a variety of other securities, broker dealers must, inter alia, pass a 6-hour Series 7 exam, while investment adviser representatives must pass a 3-hour Series 65 exam License Requirements.

108 Test questions do presuppose a knowledge of beta. For example one question imagines that a client’s $1,000,000 portfolio has an “aggressive bias toward growth stocks” and says that the portfolio’s “beta is 1.4” before asking how many S&P 500 index puts would be required to hedge the downside risk if the index is currently at $2,000. http://www.series7practiceexam.com/series-7-practice-exam-1/

109 The new Department of Labor Fiduciary Rule, 81 Fed. Reg. 20946 (April 8, 2016), “a person is a fiduciary to [an] IRA to the extent that the person engages in specified plan activities, including rendering ‘investment advice for a fee or other compensation, direct or indirect, with respect to any moneys or other property of such plan . . . [,]’” FINRA’s Rule 2111 also subjects broker dealers who recommend that clients role over their 401(k) balances to IRA accounts are subject to suitability duties. See Investment Company Institute (ICI) Frequently Asked Questions About Individual Retirement Accounts and 2013 Investment Company Fact Book at 114. The proposals in this section, including those regarding periodic individualized portfolio analysis might easily apply to Individual Retirement Accounts which accounted in 2012 for 28% of U.S. retirement accounts. Id.
participants. But the “safe harbor” provision of the statute—section 404(c)—immunizes plan sponsors from fiduciary liability for any loss, or by reason of any breach, which results from such participant’s or beneficiary’s exercise of control. To qualify for this safe-harbor protection, plans must satisfy three prerequisites:

First, the participant must have the right to exercise independent control over assets in his or her account and must in fact exercise such control. Next, the participant must be able to choose from a broad range of investment alternatives, which requires at least three investment options and the plan must permit the participant to give instructions to the plan with respect to those options once every three months. Third, the participant must be given or have the opportunity to obtain sufficient information to make informed decisions with regard to investment alternatives available under the plan.

This third “sufficient information” requirement of is particularly relevant.

The Department of Labor should issue new regulations mandating periodic disclosure of individualized participant portfolio analysis as a prerequisite for this safe-harbor immunity. While individualized analysis might have been costly in the past, the advent of fin-tech robo-advisors show that the marginal cost of providing such information is essentially zero. A host of firms including SigFig, Betterment, FutureAdvisors and Wealthfront currently provide portfolio analysis for free. The essence of this portfolio review would be to assess potential losses from diversification, beta or excessive fees mistakes. The disclosures would include estimates of how much the portfolio would have to be expected to beat the market in order to justify the diversification and or other failures. The disclosures should provide information on specific transactions which would reduce the diversification, exposure and/or fee losses. Again, the demonstrated success of SigFig and other FinTec companies underscore the feasibility of providing such information. These “alpha-tized” warnings should let participants know how often portfolios with this level of alpha deviation ended up beating the plan’s default investment

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113 The providers often will also manage your portfolio for a fee, https://www.nerdwallet.com/blog/investing/online-advisor-comparison/
This personalized information fits well with the existing requirements to give participants “the opportunity to obtain sufficient information to make informed decisions.”

Providing individualized participant portfolio analysis would also enhance disclosure of potential participant investment mistakes on the Form 5500 which plan sponsors are required to file annually with the Department of Labor. The second safe-harbor requirement that “participant must be able to choose from a broad range of investment alternatives” has been found to require a compliant plan to give participants the opportunity to diversify away most idiosyncratic risk. But the current Form 5500 disclosure does not give regulators or sponsors any way to tell whether individual participants are taking undiversified positions. Fin-tech algorithms could be easily programmed to provide aggregate information about the extent of diversification, exposure or excess fee losses and how often these losses are in fact offset by market-beating returns.

The most important recent change to the landscape of retirement investing is the “fiduciary rule” promulgated by the Department of Labor in April 2016. The rule significantly broadened the definition of who becomes a fiduciary by rendering advice regarding retirement plan investments. In particular, under the previous regulation, a person became a fiduciary by virtue of rendering investment advice only if: (1) the advice is rendered as to the value of securities or property or as to the advisability of investing in securities or property; (2) on a regular basis; (3) pursuant to a mutual agreement or understanding between the adviser and the client; (4) that the advice will serve as the primary basis for investment decisions; and (5)

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114 ERISA allows plans to establish default investments, so-called QDIAs (Qualified Default Investment Alternatives) – which must be tailored to avoid two of the three central investment mistakes (lack of diversification and non-optimal exposure to equities). Ayres & Curtis, supra note 10, at 1515, have argued for an Enhanced QDIA that would also mandate non-excessive fees. Ideally, the alpha-tized warnings would reveal how often plan participants with alpha-deviations of various magnitudes end up beating the enhanced QDIA – and that plans without a default investment announce a qualifying comparator portfolio for these disclosure purposes.

115 Another low-cost opportunity for improved personalized disclosure concern “mapping,” the default reinvestment of participant savings into a sponsor-chosen fund when a pre-existing menu option is discontinued. At the moment, plan sponsors send participants warning that, absent participant objection, investments will be mapped into the new fund—regardless of whether participants have invested in the discontinued fund. As a result of this untailored disclosure, most mapping disclosure have no relevance to most participants who overtime learn to ignore them. Ian Ayres, The Problem Of 401(k) Mapping To Dominated Funds, FORBES (2014) available at https://www.forbes.com/sites/whynot/2014/03/29/the-problem-of-401k-mapping-to-dominated-funds/#7d66d0e96974. Instead, mapping warning should only go to participants who are currently invested in the fund that is to be discontinued. The warning should disclose the expense ratio of the discontinued and the default fund to which the investments will be mapped, and whether the mapping to will increase the offsetting alpha necessary to justify diversification, exposure, or excess fee losses.

116 See I. Ayres & Q. Curtis, supra note 10, at 1490 (estimating that menu diversification failures are equivalent to just 5 basis points of lower return).

117 The form requires disclosure of aggregate plan investments which might substantially mask diversification problems (for example, if some participants are solely invested in bonds while others are solely invested in equities).

118 It might also be advisable to analogously require that broker/dealers report the extent to which their clients’ portfolios are not fully diversified.

that it will be particularized to the individual needs of the retirement investor."¹²⁰ This narrow fiduciary definition “allowed advisers comfortably to conclude that they were not acting as ERISA fiduciaries when making most investment recommendations to retail retirement clients.”¹²¹

The new rule, by contrast, uses a much more expansive definition, pulling most investment recommendations to retail retirement clients into the ambit of the rule. If an individual or entity becomes a fiduciary by rendering advice to a retirement client, the advisor will be prohibited under ERISA from accepting commissions and other common forms of third-party compensation unless the advisor qualifies for an exception like the fiduciary rule’s “Best Interest Contract” exemption.¹²² To qualify for that exemption, advisors must, among other things, accept no more than reasonable compensation and commit to acting impartially and in the customer’s best interest.¹²³ Along with the bans on transactions deemed by ERISA to be conflicted, being an ERISA fiduciary imposes other duties of prudence and exclusive benefit, the violation of which can expose the advisor to liability.¹²⁴

Proponents of the fiduciary rule argue that investment advisors tend to steer clients into funds which provide the advisors with the largest commissions, which in turn tend to be high-fee and under-diversified.¹²⁵ The Council of Economic Advisors in 2015, for example, estimated that conflicted retirement advice leads to investments which underperform low cost indices by 100 basis points, imposing an estimated $17 billion annual cost for savers in IRAs who rely on conflicted advice.

Notwithstanding the large potential gains from the rule estimated by the Obama administration, it has been highly controversial. Critics argue that it will increase the cost of providing investment advice because it increases compliance costs and potential legal liability.¹²⁶

¹²¹ Kenneth J. Laverierre and Matthew H. Behrens, The US department of labor’s final “fiduciary” rule incorporates concessions to financial service industry but still poses key challenges, 17 J. Inv. Compliance 1, 2 (2016).
¹²³ See Hugler, 231 F. Supp. 3d at 164 (noting that advisers may only qualify for the best interest contract exemption if they “enter into a written contract with the retirement investor, agreeing to: 1) acknowledge their fiduciary status, 2) commit to complying with standards of impartial conduct and to act in the customer’s “best interest,” 3) receive no more than “reasonable compensation,” 4) adopt policies and procedures reasonably designed to minimize the effect of conflicts of interest, and 5) disclose basic information about conflicts of interest and the cost of their advice.”).
¹²⁴ See Laverierre & Behrens, supra note 121, at 6.
¹²⁵ See COUNCIL OF ECONOMIC ADVISORS, THE EFFECTS OF CONFLICTED INVESTMENT ADVICE ON RETIREMENT SAVINGS (2015) (citing a variety of academic studies showing that funds sold primarily on commission tend to underperform low cost index funds, and that all else equal, funds with higher commissions receive more fund inflows, but underperform by more than funds with lower commissions).
¹²⁶ See, e.g., Jill E. Fisch, Tess Wilkinson-Ryan, and Kristin Firth, The Knowledge Gap in Workplace Retirement Investing and the Role of Professional Advisors, 66 Duke LJ 633 647 & n. 81 (2016) (summarizing the criticisms and presenting a survey which shows an important potential role for financial advisors in preventing investors from making a variety of investment mistakes); Securities Industry and Financial Markets Association, The DOL
The Trump administration, shortly after inauguration, ordered the Department of Labor to prepare new economic impact analyses and strongly implied the Department should revise or rescind the rule. The Department of Labor then delayed implementation of parts of the rule to June 9, 2017, pushed back farther other parts of the rule which had already been scheduled to come into effect later, and removed certain transition requirements. Some of the rule in fact went into effect in June 2017, but the Department has recently sought to further delay full implementation of the rule to July 1, 2019.

It is safe to say the rule faces a highly uncertain future with the current administration. We believe the problem of conflicted retirement advice is a very serious one, which has likely resulted in millions of investors unsuccessfully, and often unwittingly, chasing alpha through high-fee and under-diversified mutual funds. Nevertheless, we do see at least reasonable room for disagreement about whether the fiduciary rule is the optimal response to this problem because of its relatively high cost and potential to reduce the provision of financial advice. The fin-tech algorithms which we argued above could be required as part of the safe-harbor for 401(k) plan administrators could also be useful in the broader problem of policing conflicted retirement advice.

In particular, clear, concise, and salient estimates of the alpha needed to justify investing in high-fee funds—and how infrequently such funds deliver that alpha—might make investors more likely to reject conflicted advice to buy a high-fee or under-diversified fund. These estimates can be provided at much lower cost than those imposed by the fiduciary rule. Thus mandated robo-advising can be viewed as a relatively low cost, more libertarian substitute for parts of the fiduciary rule.

Of course, using fin-tech will also be less effective at reducing losses from conflicted advice than the fiduciary rule. The fee structures of the funds recommended by advisors are not secret, and all the robo-advisors can do is to tell investors that investing in most high fee, under-diversified funds will not payoff. When investors with relatively limited financial sophistication are confronted with (conflicted) advice to buy a fund, however, it is unclear how much even very clear disclosures about high fees and alpha can do.

The foregoing 401(k) and fiduciary reform proposals fit comfortably within the current 404(c) safe-harbor regime and definition of fiduciary under the Act and could be implemented by

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129 See Fisch, Wilkinson-Ryan, & Firth, supra note 126. As we discuss below, a more effective, albeit more restrictive, alternative would be to require investors whose first portfolio choice appears to require capturing more than a certain amount of alpha (say 100 basis points) to pass a financial sophistication test before being allowed to pursue this presumptively foolish path using the tax subsidy implicit in 401(k) and IRA funds.
enlightened DOL regulation. However, our analysis also has potential implications for more far reaching reforms to reduce the likelihood of investment error. To begin, plan sponsors might be required to assess whether their menu offerings are alpha-justified. For example, the choice to offer corporate stock is likely to harm participants (whose human capital is often already over exposed to their employer’s fate) by needlessly increasing idiosyncratic risk if participants invest too large a percentage of their saving portfolio in this single equity. Offering undiversified menu offerings is problematic if the plan sponsor has not considered the offsetting alpha necessary to justify the probable failures of participant diversification. Similarly, plan sponsors should be asked whether menu offerings are alpha justified given the tendency of participants to naively diversify by putting some of their savings in every menu offering.\footnote{Shlomo Benartzi & Richard H. Thaler, \textit{Naive Diversification Strategies in Defined Contribution Saving Plans}, 91 AM. ECON. REV. 79 (2001).} Sponsors should consider whether their menus include funds are “dominated” by other menu options, and if so whether the fund’s presumptive losses might plausibly be justified by offsetting expected alpha.\footnote{As defined by Ayres & Curtis, a menu offering is dominated if no rationale investor would invest funds in this option given the other option available on the menu. \textit{See I. Ayres & Q. Curtis, supra note 10, at 1504. A high-cost S&P index option would be an example of a dominated fund if a lower-cost S&P index was also available. Ayres & Curtis found that nearly half of 401(k) plans that they analyzed included dominated funds in their menus and that these dominated funds garnered 11.5 percent of plan assets. \textit{Id.} at 1506.}}

In addition to providing participants with enhanced, individualized information about their portfolios, ERISA might go further and require participants to pass a “Retirement Plan Investment Sophistication” test before investing in portfolio that needed an offsetting alpha of over 1% annually. This testing requirement is an example of an altering rule that reduces the likelihood of error by requiring individuals to demonstrate actual knowledge of the issues related to opt out before they can deviate from the status quo.\footnote{Ayres, \textit{supra} note 23, at 2076.} Train and test altering has been deployed in other high-stakes settings (such as student loans and human subjects approval)\footnote{The 2008 Higher Education Opportunity Act encourages institutions administering student loan programs to use “interactive programs that test the borrower’s understanding of the terms and conditions of the borrower’s loans . . . using simple and understandable language and clear formatting.” 20 U.S.C. § 1092(l)(1)(B) (2012) (emphasis added). Health Insurance Portability and Accountability Act (HIPAA) regulations require researchers to train and test on the requisite privacy protection before they can access personal health information. 45 C.F.R. § 164.530(b) (2010).} and has been recommended for testing securities sophistication.\footnote{A 2009 \textit{Forbes} article suggested that sophisticated investors “should be required to meet minimum qualifications, attend a few educational classes and pass a basic test of knowledge of the markets.” John E. Girouard, \textit{The Sophisticated Investor Farce}, FORBES (Mar. 24, 2009, 12:30 PM), http://www.forbes.com/2009/03/24/accredited-investor-sec-personal-finance-financial-advisor-network-net-worth.html [http://perma.cc/Z66B-WW6B].} Participants who demonstrate by passing the test that they are aware of the kinds of tradeoffs at the core of our analysis would be free to seek alpha in ways that exposed them to some mixture of diversification, exposure or excess fee losses. But the vast majority of shareholders would be restricted from pursuing non-standard investment strategies.\footnote{The possibility of a sophistication test is also explored by I. Ayres & Q. Curtis, \textit{supra} note 10, at 1515, albeit without testing participants’ knowledge of alpha tradeoffs.} Requiring IRA investors to pass a test concerning alpha tradeoffs before putting their savings in high-fee, under-diversified funds would be more
effective in reducing the effect of conflicted investment advice than required disclosure of the implicit alpha bet via a robo-advisor, which was suggested above. A policy based on testing—while defensible both as consumer protection and conserving the taxpayer subsidy implicit in IRAs—might well be politically infeasible, however. As with the fiduciary rule, it would engender strong opposition from parties who benefit from the current system, and it might prove far more unpopular with the public than the fiduciary rule.

Our finding that the size of off-setting alpha can vary substantially overtime also suggests that plans might also institute an automatic remapping of participant portfolios. A portfolio that, for example, starts bearing more costly idiosyncratic risk during the time of a crisis might by default be mapped into a portfolio that requires less of an offsetting alpha unless the participant owner either passes the sophistication test (and thereby gains the right to make alpha opportunity bets) or, before the mapping, the participant self-directs the investments to a portfolio with a sufficiently lower diversification/exposure/excess fee losses.

V. CONCLUSION

Instead of prohibiting financial advisors, and other fiduciaries from recommending or allowing clients to pursue “seeking alpha” strategies, we have proposed making sure that the fiduciaries understand how much alpha is required to offset specific kinds of losses. Failures to minimize excess fees, diversification and exposure losses can be justified by sufficient expectations that a portfolio will generate above market returns. This Article has estimated how much alpha is necessary to offset particular kinds of portfolio losses – from taking on inefficient idiosyncratic risk (diversification loss), inefficient systemic risk (exposure loss) or inefficient excess fees. We find that diversification loss is likely higher during crisis periods, both because systemic risk tends to be higher and because idiosyncratic risk tends to be higher. Requiring that fiduciaries understand these alpha tradeoffs might reduce both Type I and Type II errors. Some fiduciaries will refrain from pursuing alpha investment opportunities when they learn that the benefit of the expected alpha is outweighed by the expected losses. Other fiduciaries who are now deterred from alpha opportunities may discover that the expected benefits of some of these investments outweigh their costs.

It is not necessarily stupid to put all of your retirement eggs in one basket. But most people who do so are making a grave mistake. This article integrates both of these maxims by acknowledging that alpha opportunities can render non-standard investment strategies rational and showing that the required offsetting alpha to justify diversification, exposure and excess fee losses are often surprising large.

136 Reducing Type II errors might exacerbate the problem of “common ownership” if firms that compete in product or service markets become increasing owned by an increasing group of diversifying shareholders. Einer Elhauge, Horizontal Shareholding, 129 Harv. L. Rev. 1267 (2016); David Gilo, Yossi Moshe & Yossi Spiegel, Partial cross ownership and tacit collusion, 37 Rand J. Econ. 81 (2006); Erik P. Gilje, Todd Gormley & Doron Levit, The Rise of Common Ownership (working paper 2017). However, the potential anti-competitive effect of inducing increased diversification can be remedied by other government interventions. See, e.g., Eric Posner, Fiona-Scott Morton & Eric Weyl, A Proposal to Limit AntiCompetitive Power of Institutional Investors (working paper 2016); Ian Ayres & Stephen F. Ross, “Pro-competitive Executive Compensation” as a Condition for Approval of Mergers that Simultaneously Exploit Consumers and Enhance Efficiency, 19 CANADIAN COMPETITION RECORD 18 (Spring 1998).